Homework #2

Due Tuesday, September 9 Wednesday, September 10 in Gradescope by 11:59 pm ET

READ | Sections 1.1, 1.2 in Richmond&Richmond

WATCH Video 2: A Union Proof (19:37)

[Found on moodle site]

WRITE AND SUBMIT solutions to the following problems. ALWAYS justify your claims.

Problem 1. (4 points) Section 1.1, #2

Which has the larger cardinality? The set of letters in the word MISSISSIPPI or the set of letters in the word FLORIDA?

Problem 2. (24 points) Section 1.1, #10(b(iv-ix))

For each $k \in \{1, 2, ..., 20\}$, let $D_k = \{x \mid x \text{ is a prime number that divides } k\}$.

Let $\mathcal{D} = \{D_k | k = 1, 2, \dots, 20\}$. True or false (and briefly justify):

(iv)
$$\varnothing \in \mathcal{D}$$

$$(v) \varnothing \subsetneq \mathcal{D}$$

(vi)
$$5 \in \mathcal{D}$$

(iv)
$$\varnothing \in \mathcal{D}$$

(vii) $\{5\} \in \mathcal{D}$

$$(v) \varnothing \subsetneq \mathcal{D}$$
$$(viii) \{4,5\} \in \mathcal{D}$$

(ix)
$$\{\{3\}\}\subseteq \mathcal{D}$$

Problem 3. (18 points) Section 1.2, #4(a,b,d,h,m,n)

Let U be the set of 52 cards in a standard deck. Let S, D, A, K be the sets of spades, diamonds, aces, and kings, respectively. Say which cards belong to each set below, and find the cardinality of each set. (And briefly justify.)

(a)
$$A \cap D$$

(b)
$$S \cap D$$

(d)
$$(A \cup K) \cap (S \cup D)$$

(h)
$$K \cap [(S \cup D)^c]$$
 (m) $S \setminus K$

(m)
$$S \setminus K$$

(n)
$$K \setminus S$$

Problem 4. (6 points) Section 1.2, #7(b), first two parts

Determine whether the sets in this collection are mutually disjoint. Also determine whether the collection is nested. (And of course, briefly justify everything!)

$$\mathcal{B} = \left\{ \left(-\frac{1}{n}, n \right) \middle| n \in \mathbb{N} \right\}$$

Problem 5. (16 points) Section 1.2, #7(b), last two parts

Find the union of the sets in the collection \mathcal{B} above. Also find the intersection of the sets in \mathcal{B} . In both cases, say what the set is and also **prove** the equality of sets you are claiming.

Problem 6. (8 points) Section 1.2, #8(d,e)

Determine whether the following statements are true or false, and briefly justify.

(d): $\mathcal{B} \subseteq \mathcal{A}$, where \mathcal{B} is as in the previous problem, and $\mathcal{A} = \left\{ \left(\frac{1}{n}, n+1 \right) \mid n \in \mathbb{N} \right\}$

(e):
$$\mathcal{C} \subseteq \mathcal{D}$$
, where $\mathcal{C} = \{(n, \infty) \mid n \in \mathbb{N}\}$ and $\mathcal{D} = \{(x, \infty) \mid x \in \mathbb{R}\}$

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

 $\begin{array}{ll} \mbox{Mondays} & 2:00-3:30\mbox{pm} \\ \mbox{Tuesdays} & 1:45-3:15\mbox{pm} \\ \mbox{Fridays} & 1:00-2:00\mbox{pm} \end{array}$

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri 10:00am-noon Tue/Thu 1:30-4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00-7:30 pm	Aaron Cordoba
Mondays	7:30-9:00pm	John Lim
Tuesdays	6:00-7:30 pm	Aaron Cordoba
Tuesdays	7:30-9:00pm	Gretta Ineza
Wednesdays	7:30-9:00pm	John Lim
Thursdays	6:00-7:30 pm	Gretta Ineza

Also, you may email me any time at ${\tt rlbenedetto@amherst.edu}$