Homework #15

Due **WEDNESDAY, November 5** in Gradescope by **11:59 pm ET**

READ Section 8.1 in Richmond&Richmond

WATCH Lecture Videos A–H [Found on moodle site]

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (12 points) Section 8.1, #2
Give the first five terms, and the tenth term, of each of the following sequences:

(a) $(a_n)_{n=-3}^{\infty}$ where $a_n = (-2)^n$
(b) $(b_n)_{n=5}^{\infty}$ where $b_n = 12n - 121$
(c) $(c_n)_{n=-1}^{\infty}$ where $c_n = 2n^2 - n + 1$

[**Note**: These sequences do not start with $n = 1$. Don't give the terms corresponding to $n = 1, 2, 3, 4, 5, 10$, but rather the first five terms, and the tenth term, of each.]

Problem 2. (15 points) Section 8.1, #6(a, c, d)
Let $(a_n)_{n=1}^{\infty}$ be the real sequence given by $a_n = 2n - 5$. For each function $f : \mathbb{N} \to \mathbb{N}$ below, find the sequence $(b_n)_{n=1}^{\infty}$ given by $b_n = a_{f(n)}$. Determine whether or not $(b_n)_{n=1}^{\infty}$ is a subsequence of $(a_n)_{n=1}^{\infty}$. (And briefly explain why, of course.)

(a) $f(n) = n^2$
(c) $f(n) = \lfloor \frac{n}{2} \rfloor$
(d) $f(n) = |2n - 7|$

Problem 3. (15 points) Section 8.1, #7, variant
Prove that a real sequence $(a_n)_{n=1}^{\infty}$ is both geometric and arithmetic if and only if it is constant.

[A sequence $(a_n)_{n=1}^{\infty}$ is said to be *constant* if for all m, n, we have $a_m = a_n$.]

Problem 4. (12 points) Section 8.1, #11, variant
Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers, and let $(b_n)_{n=1}^{\infty}$ be a strictly increasing arithmetic sequence of positive integers, so that $(a_{b_n})_{n=1}^{\infty}$ is a subsequence of $(a_n)_{n=1}^{\infty}$.

(a) If $(a_n)_{n=1}^{\infty}$ is an arithmetic sequence, prove that the subsequence $(a_{b_n})_{n=1}^{\infty}$ is also arithmetic.
(b) If $(a_n)_{n=1}^{\infty}$ is a geometric sequence, prove that the subsequence $(a_{b_n})_{n=1}^{\infty}$ is also geometric.

Problem 5. (16 points) Section 8.1, #12, variant
Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers, and let $(b_n)_{n=1}^{\infty}$ be a strictly increasing geometric sequence of positive integers, so that $(a_{b_n})_{n=1}^{\infty}$ is a subsequence of $(a_n)_{n=1}^{\infty}$.

If $(a_n)_{n=1}^{\infty}$ is a non-constant arithmetic sequence, prove that the subsequence $(a_{b_n})_{n=1}^{\infty}$ is definitely **not** arithmetic.

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406): **NONE THIS WEEK**

~~Mondays~~	~~2:00–3:30pm~~	**Cancelled Monday, November 3**
~~Tuesdays~~	~~1:45–3:15pm~~	**Cancelled Tuesday, November 4**
~~Fridays~~	~~1:00–2:00pm~~	**Cancelled Friday, November 7**
~~or by appointment.~~		**No appointments this week**

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`