

Homework #14Due **Friday, October 31** in Gradescope by **11:59 pm ET****READ** Section 6.2 in Richmond&Richmond**WATCH** Video 14: Image and Inverse Image (29:13) [Found on moodle site]**WRITE AND SUBMIT** solutions to the following problems. **ALWAYS** justify your claims.**Problem 1.** (15 points) Section 6.2, #4(a,b)For each of the following functions $f : A \rightarrow B$, decide whether or not it is invertible. If it is, find the inverse function. (And as always, make sure to prove all of your claims.)

(a): $A = B = \mathbb{R}$, and $f(x) = \frac{5x - 1}{3}$

(b): $A = \mathbb{R}$, $B = [-4, \infty)$, and $f(x) = (x - 3)^2 - 4$

Problem 2. (17 points) Section 6.2, #4(c), variant

Define $f : \mathbb{Z} \rightarrow \mathbb{N}$ by $f(n) = \begin{cases} 2n & \text{if } n \geq 1, \\ -2n + 1 & \text{if } n \leq 0. \end{cases}$

Prove that f is invertible, with inverse function $f^{-1} : \mathbb{N} \rightarrow \mathbb{Z}$ given by

$$f^{-1}(m) = \begin{cases} m/2 & \text{if } m \text{ is even,} \\ -(m-1)/2 & \text{if } m \text{ is odd.} \end{cases}$$

(You may assume that $f : \mathbb{Z} \rightarrow \mathbb{N}$ and $f^{-1} : \mathbb{N} \rightarrow \mathbb{Z}$ as above are indeed both functions; but otherwise, as always, make sure to prove all of your claims.)**Problem 3.** (17 points) Section 6.2, #6(a,b), variantLet $g : \mathbb{R} \rightarrow \mathbb{Z}$ be the *ceiling function* $g(x) = \lceil x \rceil$, defined to be the smallest integer that is greater than or equal to x .

(a) Prove that the image $g([-\pi, \pi])$ is $g([-\pi, \pi]) = \{-3, -2, -1, 0, 1, 2, 3, 4\}$.

(b) Prove that the inverse image $g^{-1}(\{-2, -1\})$ is $g^{-1}(\{-2, -1\}) = (-3, -1]$.

Problem 4. (22 points)Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = 2x^2 - 7$. Prove that:

(a) $f^{-1}([2, 5)) = \left(-\sqrt{6}, -\frac{3}{\sqrt{2}}\right] \cup \left[\frac{3}{\sqrt{2}}, \sqrt{6}\right)$

(b) $f^{-1}((-10, -5]) = [-1, 1]$

(c) $f((-3, 2)) = [-7, 11)$

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays 2:00–3:30pm

Tuesdays 1:45–3:15pm

Fridays 1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri 10:00am–noon

Tue/Thu 1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays 6:00–7:30pm **Aaron** Cordoba

Mondays 7:30–9:00pm **John** Lim

Tuesdays 6:00–7:30pm **Aaron** Cordoba

Tuesdays 7:30–9:00pm **Gretta** Ineza

Wednesdays 7:30–9:00pm **John** Lim

Thursdays 6:00–7:30pm **Gretta** Ineza

Also, you may email me any time at rlbenedetto@amherst.edu