

Homework #13Due **Tuesday, October 28** in Gradescope by **11:59 pm ET****READ** Sections 6.1, 6.2 in Richmond&Richmond**WATCH** Video 13: Restriction of a Function (16:43) [Found on moodle site]**WRITE AND SUBMIT** solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (18 points) Let $X = \mathbb{R} \setminus \{5\}$ and $Y = \mathbb{R} \setminus \{2\}$. Define $f : X \rightarrow Y$ by $f(x) = \frac{2x-3}{x-5}$.

- (a) (5 points): Prove that f is defined, i.e., for every $x \in X$, the value $f(x)$ is defined and is in Y .
- (b) (5 points): Prove that f is one-to-one.
- (c) (8 points): Prove that f is onto.

Problem 2. (5 points)Let $f : A \rightarrow B$ be a function, and let $C \subseteq A$ be a subset. Define $g : C \rightarrow A$ by $g(x) = x$.Prove that $f|_C = f \circ g$ [Recall that $f|_C$, called the *restriction* of f to C , is the function $f|_C : C \rightarrow B$ given by $f|_C(x) = f(x)$ for $x \in C$.]**Problem 3.** (10 points)Let $A \neq \emptyset$ be a nonempty set. Define $f : A \rightarrow \mathcal{P}(A)$ by $f(x) = \{x\}$.

- (a) (5 points): Prove that f is one-to-one.
- (b) (5 points): Prove that f is **not** onto.

Problem 4. (16 points)Let B be a set, and define $g : \mathcal{P}(B) \rightarrow \mathcal{P}(B)$ by $g(S) = B \setminus S$.

- (a) (4 points): Prove that g is indeed a function.
- (b) (5 points): Prove that for every $S \in \mathcal{P}(B)$, we have $g \circ g(S) = S$.
- (c) (7 points): Prove that g is bijective. [**Suggestion:** Use part (b) above.]

Problem 5. Section 6.2, #14 (12 points)Prove Theorem 6.2.8(b): Let $f : A \rightarrow B$ be a function, and let $C, D \subseteq A$ be subsets. Prove that $f(C \cup D) = f(C) \cup f(D)$ **Problem 6.** (15 points)Define $h : [-4, 4] \rightarrow [3, 7]$ by $h(x) = 3 + \sqrt{16 - x^2}$. Prove that

- (a) (5 points): h is indeed a function.
- (b) (6 points): h is onto.
- (c) (4 points): h is **not** one-to-one.

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays 2:00–3:30pm

Tuesdays 1:45–3:15pm

Fridays 1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri 10:00am–noon

Tue/Thu 1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays 6:00–7:30pm **Aaron** Cordoba

Mondays 7:30–9:00pm **John** Lim

Tuesdays 6:00–7:30pm **Aaron** Cordoba

Tuesdays 7:30–9:00pm **Gretta** Ineza

Wednesdays 7:30–9:00pm **John** Lim

Thursdays 6:00–7:30pm **Gretta** Ineza

Also, you may email me any time at rlbenedetto@amherst.edu