Homework #12

Due **Friday, October 24** in Gradescope by **11:59 pm ET**

READ $\boxed{\text{Section 6.1 in Richmond\&Richmond}}$

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (20 points) Section 6.1, #11(a,b)
Define $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ by $f(n) = \{n, 2n, 3n, \ldots\} = \{kn \,|\, k \in \mathbb{N}\}$

Define $g : \mathcal{P}(\mathbb{N}) \to \mathbb{N}$ by $g(A) = \begin{cases} 1 & \text{if } A = \varnothing, \\ \min(A) & \text{if } A \neq \varnothing \end{cases}$

- (a): Is f injective? Is f surjective? Prove your answers.
- (b): Is g injective? Is g surjective? Prove you answers.

[Recall that min A denotes the smallest element of the nonempty subset A of $\mathbb{N}$. You may assume, without proof, that both f and g are functions.]

Problem 2. (12 points) Section 6.1, #11(c,d), variant
For f and g as in the previous problem:

- (c): Prove that for every $n \in \mathbb{N}$, we have $g \circ f(n) = n$.
- (d): Give three different examples of sets $A \in \mathcal{P}(\mathbb{N})$ such that $f \circ g(A) \neq A$.
  (And of course, verify this inequality for each of your three examples.)

Problem 3. (10 points) Section 6.1, #13
Prove that composition of functions is associative. That is:
For any functions $f : A \to B$, $g : B \to C$, and $h : C \to D$, prove that $h \circ (g \circ f) = (h \circ g) \circ f$

Problem 4. (12 points) Section 6.1, #16
Prove Theorem 6.1.14(b): for any functions $f : A \to B$ and $g : B \to C$, if f and g are both onto, then $g \circ f : A \to C$ is also onto.

Problem 5. (15 points) Section 6.1, #17(a)
Let $f : A \to B$ and $g : B \to C$ be functions.

- (i): If $g \circ f$ is onto, prove that g is onto.
- (ii): Show the converse of (i) fails. That is, give examples of functions f, g as above for which g is onto, but $g \circ f$ is not onto. (And prove your claims, of course.)

Problem 6. (15 points) Section 6.1, #17(b)
Let $f : A \to B$ and $g : B \to C$ be functions.

- (i): If $g \circ f$ is one-to-one, prove that f is one-to-one.
- (ii): Show the converse of (i) fails. That is, give examples of functions f, g as above for which f is one-to-one, but $g \circ f$ is not one-to-one. (And prove your claims, of course.)

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`