Homework #11

Due **Tuesday, October 21** in Gradescope by **11:59 pm ET**

READ Section 6.1 in Richmond&Richmond

WATCH 1. Video 11: Proving the Well-Ordering Principle (Optional) (12:04)

2. Video 12: Onto and One-to-One (27:04) [Found on moodle site]

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (20 points) Section 6.1, #3
Let $S = \{1, 2, 3, 4\}$. For each of the following subsets of $S \times S$, determine whether it is a function. For each one that is a function, find its range, determine whether it is one-to-one, and determine whether it is onto.

(a): $\{(1, 2), (2, 1), (3, 4), (4, 3)\}$ (b): $\{(1, 1), (3, 1), (2, 1), (4, 1)\}$

(c): $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$ (d): $\{(1, 3), (2, 4), (3, 3), (4, 3)\}$

(e): $\{(1, 3), (2, 1), (3, 2)\}$

[Note: as always, don't forget to justify every claim you make!]

Problem 2. (15 points) Section 6.1, #4(a)
Let $f : [0, \infty) \to (0, 1]$ by $f(x) = \dfrac{1}{x+1}$. You may assume that f is a function. Determine whether it is one-to-one, onto, neither, or both. If it is not onto, determine its range.
As always, prove all of your claims. (Some proofs are very short!)

Problem 3. (12 points) Section 6.1, #4(b)
Let $s : \mathbb{R} \to \mathbb{R}$ by $s(x) = \sin x$. You may assume that s is a function. Determine whether it is one-to-one, onto, neither, or both. If it is not onto, determine its range.
As always, justify all of your claims. (Some justifications are very short! You may assume standard facts from high school trigonometry.)

Problem 4. (18 points) Section 6.1, #4(c)
Let $g : \mathbb{N} \to \mathbb{N}$ by $g(n) = \lfloor (n^2 + 3)/n \rfloor$. You may assume that g is a function. Determine whether it is one-to-one, onto, neither, or both. If it is not onto, determine its range.
As always, prove all of your claims. (Some proofs are very short! Remember that the *floor function* $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

Problem 5. (15 points) Section 6.1, #5(a)
Consider the function $g : \{n \in \mathbb{N} \mid n \geq 100\} \to \mathbb{N}$ by $g(n) =$ the sum of the digits of n. Determine whether g is one-to-one, onto, neither, or both. If it is not onto, determine its range.
As always, prove all of your claims.

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`