### Homework #10

Due Friday, October 17 in Gradescope by 11:59 pm ET

**READ** Section 6.1 in Richmond&Richmond

WATCH Video 10: Exam 1 Proof Review (30:06) [Found on moodle site]

WRITE AND SUBMIT solutions to the following problems. ALWAYS justify your claims.

### Problem 1. (16 points)

Prove that 
$$\bigcap_{t \in (2,6)} [0, 2t + 5) = [0, 9]$$

#### Problem 2. (10 points)

Prove that there is a unique real number  $c \in \mathbb{R}$  such that for all  $t \in \mathbb{R}$ , we have ct - 3c + 12 = 4t. [Don't forget to prove **both** the existence and the uniqueness parts.]

#### Problem 3. (14 points)

Prove that for every  $y \in [-2, 2]$ , there is some  $x \in [1, 3]$  such that  $\frac{6}{x} - 4 = y$ .

#### Problem 4. (11 points)

Define a sequence  $c_1, c_2, c_3, \ldots$  of real numbers by:

$$c_1 = \frac{1}{2}$$
, and for every  $n \ge 1$ ,  $c_{n+1} = c_n - c_n^2$ .

Use mathematical induction to prove that for all  $n \in \mathbb{N}$ , we have  $0 < c_n < 1$ .

## Problem 5. (14 points) Section 2.2, #12

Use mathematical induction to prove that for all integers  $n \geq 2$ , we have

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

# Questions? You can ask in class or in:

## My (Drop-In) Office Hours (SMUD 406):

Mondays 2:00–3:30pm Cancelled Monday, October 13 Tuesdays 1:45–3:15pm Cancelled Tuesday, October 14

Fridays 1:00–2:00pm

or by appointment.

This week only: Wednesday, Oct 15 1:00–2:30pm

## Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri 10:00am-noon Tue/Thu 1:30-4:30pm

### Math Fellow Drop-in Hours (SMUD 006):

Mondays 6:00-7:30pm **Aaron** Cordoba John Lim Mondays 7:30-9:00pm **Aaron** Cordoba Tuesdays 6:00-7:30 pmGretta Ineza Tuesdays 7:30–9:00pm John Lim Wednesdays 7:30–9:00pm Thursdays 6:00-7:30 pmGretta Ineza

**WARNING**: Office hours, QCenter hours, and Fellow hours are generally cancelled on vacation days!

In particular, for Fall Break (Monday, Oct 13 and Tuesday, Oct 14):

- My Monday/Tuesday office hours are cancelled (but I'm adding hours on Wednesday, Oct 15, 1–2:30pm).
- Allison's Monday/Tuesday Drop-in hours are cancelled.
- Monday evening Fellow Drop-in hours are cancelled, but:
- Aaron will hold his Tuesday 6–7:30pm Drop-in hours