Math 220, Section 03, Fall 2025 Professor Rob Benedetto

What you need to know for the Final Exam

The exam (9am–noon on Thursday, December 18, in SMUD 205) will cover the whole semester. Below is a list of most of the topics covered since Exam 2. By combining this handout with the review handouts from the other two exams, you will have a summary of the full semester. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

- Section 8.3: Limits of real sequences. The ε-N definition of $\lim_{n\to\infty} a_n = L$, that the sequence $(a_n)_{n=1}^{\infty}$ converges, i.e., that its limit exists. What it means for a sequence to diverge. Uniqueness of the limit if it exists (Theorem 8.3.4). Diverging to infinity or to minus infinity (Definiton 8.3.5). Theorem 8.3.9, about the convergence and limits of the sequences $(a_n + b_n)$, and (ka_n), and $(a_n b_n)$, **provided** we know that the sequences (a_n) and (b_n) converge.

- Section 8.4: Sequences being bounded above, bounded below, or just bounded. The Monotone Sequence Theorem (Theorem 8.4.1): any bounded monotone sequence converges. Theorem 8.4.2 (if $a_n \leq b_n$ and $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then $L \leq M$) and the Squeeze Law (Theorem 8.4.3), and especially the methods that go into their proofs. Theorem 8.4.4 (but *not* its proof), that every bounded real sequence has a convergent subsequence.

- Section 6.3: Cardinality. Meaning of $|A| = |B|$: that there is a bijective function $f : A \to B$. Countably infinite, countable, uncountable. Theorems 6.3.3, 6.3.7 and 6.3.9, and Corollaries 6.3.8 and 6.3.10, which together say (among other things) that $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$, and $\mathbb{Q}$ are all countable. Understand their proofs, too. The Hilbert Hotel, just for general understanding.

  (Note: In the practice materials for Midterm Exam 1, I had said you don't need to know about cardinality. But now you do.)

- Section 6.4: Ordering Cardinals. Meaning of $|A| \leq |B|$: that there is an injective function $f : A \to B$. Statement (but not proof) of the Schröder-Bernstein Theorem. Theorems/Corollaries 6.4.5–6.4.10, about countable and uncountable sets. Theorem 6.4.11, that $|A| < |\mathcal{P}(A)|$.

Some things you don't need to know

- Throughout: all references to relations other than functions.

- Section 8.3: Theorem 8.3.8, that if the sequence (a_n) diverges to ∞ or to $-\infty$, then the sequence $(1/a_n)$ converges to 0.

- Supremum (least upper bound) and infimum (greatest lower bound) of a set $S \subseteq \mathbb{R}$. The Completeness Axiom for $\mathbb{R}$, that every nonempty subset $S \subseteq \mathbb{R}$ has a supremum.

- Section 8.4: The proof of Theorem 8.4.4. The statements *and* proofs of Theorem 8.4.5 and Corollary 8.4.6.

- Section 8.5: You don't need to know anything in this section.

- Section 6.3: Lemma 6.3.4, Theorem 6.3.5, and Theorem 6.3.6.

- Section 6.4: The *proof* of the Schröder-Bernstein Theorem (Theorem 6.4.3).

- Section 6.4: Corollary 6.4.4.

Tips

- All the tips from the first two exams still apply. Pull out the review handouts from those exams and read those tips carefully.
- Get started *immediately* on your studying. Obviously, you have other classes; but it's important to spend at least 20 minutes (and preferably more) *every* day on math. Otherwise, it slips away from the memory a lot faster than you might think. With five days to go, make sure to put in at least an hour a day.

  Then, as the date of the exam approaches, start studying more in earnest. Make sure to do *lots* of practice problems — both the new ones, *and* the old ones. Do some old homework problems, too. Plan ahead; you will need to start working *days* in advance to make that happen.

  And the night before the exam, get a good night's sleep.
- Arrive to the exam early. (We'll be in our regular classroom: SMUD 205.) Plan ahead to take care of getting to the building, making a last restroom run, getting yourself settled, etc., so that you are completely ready to go at least a few minutes **before** the 9:00am start time.
- You will have three hours, but the exam will be less than three times as long as either of the (50-minute) midterms. So don't rush, but *do* work methodically through the problems.
- For proof problems, work out (and probably even write down) precisely what the statement you're trying to prove is. (E.g., not just "f is one-to-one," but "$\forall x, y \in S$ with $f(x) = f(y)$, we have $x = y$.")

  Then use that precise statement to design the outline of your proof, including first and last lines (like "Given $x, y \in S$ with $f(x) = f(y)$" and "so $x = y$").

  Now start trying to prove it; and *only now* take a look at what the hypotheses were.

  Use scratchwork if you're stuck. Some problems (especially those involving $\exists$ anywhere, e.g. proving a function is onto, or an ε-N limit proof) may require some scratchwork to work backwards from the desired conclusion.
- As on the midterms, you may bring one sheet of handwritten notes to the exam: standard size (8.5x11" or so), and you may use both sides of the sheet. (Again, it may **not** be a printout. You must hand-write it yourself on paper with a pen or pencil.) Take some real time to prepare this "cheat sheet." As I said for the midterms, doing that preparation, especially writing things out yourself on paper, is itself a great study technique.