Math 220, Section 01, Fall 2026 Professor Rob Benedetto

What you need to know for Midterm Exam 2

The exam (in class, Friday, November 13) will cover the portions of Chapters 3, 6, and 8 of Richmond and Richmond that we discussed, up to and including Section 8.3. Of course, you are also responsible for all of the material from Chapters 1 and 2 required for Exam 1, plus Section 2.3 (the pigeonhole principle), although the earlier material will not be a primary focus of Exam 2. The following is a list of most of the topics covered since Exam 1. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

You may bring one standard size (8.5x11") "cheat sheet" of notes to the exam

You MAY use both sides of the sheet of paper for your cheat sheet.

You must HAND-WRITE your own cheat sheet directly on paper. No printouts.

- Section 3.1: Multiples and divisibility of integers. The notation $a|b$ and $a \nmid b$. Theorems (and Corollaries) 3.1.3–3.1.8; make sure you understand their proofs, as well. The Well-Ordering Principle (from class). The Division Algorithm (Theorem 3.1.9). Prime and composite integers. Theorems 3.1.11 and 3.1.12. Greatest common divisor, or gcd, of two integers. Relatively prime.
- Section 3.3: The Fundamental Theorem of Arithmetic (Theorem 3.3.2); know its statement (including existence *and* uniqueness), and have at least a rough idea of the proof. Corollary 3.3.3, which rephrases divisibility in terms of prime factorization.
- Section 6.1: Functions. The defining property of a function $f : A \to B$, that:

  $$\forall\, a \in A,\ \exists!\, b \in B \text{ s.t. } f(a) = b.$$

  From class (but not the book): f is *defined* (i.e., for every $a \in A$, some b exists) and *well-defined* (i.e., for every $a \in A$, b is unique). Many little terms: *domain*, *codomain* (or *target*, as I called it) *image* (or *range*), "value," "mapped to," etc. The identity function on A, $\mathrm{id}_A : A \to A$. Surjective, or onto. Injective, or one-to-one (or 1-1). Bijective, or being a one-to-one correspondence. Composition of two functions (use definition from class, where $f : A \to B$ and $g : B \to C$ compose to give $g \circ f : A \to C$). Equality of functions (Definition 6.1.12); **make sure** you know how to prove two functions are equal. Theorem 6.1.14, on injectivity and surjectivity with compositions. The *restriction* of a function (to a subset of its domain).
- Section 6.2: Inverse functions. The definition (from class) of a function $f : A \to B$ being invertible: that there exists a function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$ and $g \circ f = \mathrm{id}_A$; this is what Theorem 6.2.4 says. (And g is called the inverse of f, denoted f^{-1}.) Theorem 6.2.2, that f is invertible iff f is bijective. Theorem 6.2.3, with basic properties of invertible functions and inverses. Theorem 6.2.6, about $(g \circ f)^{-1}$. For $D \subseteq C$, the inverse image $f^{-1}(D) \subseteq A$, even (or perhaps especially) when f is not invertible. Theorems 6.2.8(a–c) and 6.2.9(a–c), about images and inverse images of unions and intersections.
- Section 8.1: (Infinite) sequences. The definition and language, the notation $(a_n)_{n=1}^{\infty}$, and what it means for two sequences to be equal. Change of index. Geometric sequences and arithmetic sequences. (Strictly) increasing and (strictly) decreasing sequences. Subsequences.
- Section 8.3: Limits of real sequences. The ε-N definition of $\lim_{n\to\infty} a_n = L$, that the sequence $(a_n)_{n=1}^{\infty}$ converges, i.e., that its limit exists. What it means for a sequence to diverge. Uniqueness of the limit if it exists (Theorem 8.3.4). Diverging to infinity or to minus infinity (Definiton 8.3.5). Theorem 8.3.9, about the convergence and limits of the sequences $(a_n + b_n)$, and (ka_n), and $(a_n b_n)$, **provided** we know that the sequences (a_n) and (b_n) converge.

Some things you don't need to know

The following are various results and concepts that you are allowed to use if you wish, but that you will not need; I will design the exam not expecting you to know them.

- Section 3.1: Least common multiple (lcm).
- Section 3.2: You are not responsible for anything in this section, much of which we skipped.
- Section 3.3: All results from Corollary 3.3.4 onward. Pythagorean triples.
- Section 6.1: The super-official definition of a function as a relation (i.e., set of ordered pairs).
- Section 6.1: The final paragraph of this section, on increasing and decreasing, order preserving and reversing, etc.
- Section 6.2: Inverses of *relations*. (We only covered inverses of *functions*.)
- Section 6.2: Part (d) of Theorem 6.2.8, and part (d) of Theorem 6.2.9.
- Section 8.1: Finite sequences. Alternating sequences. The harmonic sequence. Bounded sequences. The book's MOD notation.
- Section 8.3: Theorem 8.3.8, that if the sequence (a_n) diverges to ∞ or to $-\infty$, then the sequence $(1/a_n)$ converges to 0.

Tips

- All the tips from the first exam still apply. Pull out the review handout from that exam and read those tips carefully.
- Know how to structure proofs; in order to do that, know the definitions and theorems precisely. Know *exactly* what a proof of $a|b$, or f is one-to-one, or f is onto, or $\lim_{n\to\infty} a_n = L$, etc. look like. Again, all the tips from Exam 1 apply — each of the new definitions is a statement involving $\forall$ and/or $\exists$ and that sort of thing, so you need to make sure you can prove them.
- As on Exam 1, you may bring one sheet of handwritten notes to the exam: standard size (8.5x11" or so), and you may use both sides of the sheet. (Again, it may **not** be a printout. You must hand-write it yourself on paper with a pen or pencil.) Take some real time to prepare this "cheat sheet." As I said before Exam 1, doing that preparation, especially writing things out yourself on paper, is itself a great study technique.