Math 220, Section 03, Fall 2025 Professor Rob Benedetto

**What you need to know for Midterm Exam 1**

The exam (in class, Monday, October 6) will cover Chapter 1 (excluding Section 1.3) and the first two sections of Chapter 2 of Richmond and Richmond. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

**You may bring one standard size (8.5x11") "cheat sheet" of notes to the exam**

**You MAY use both sides of the sheet of paper for your cheat sheet.**

**You must HAND-WRITE your own cheat sheet directly on paper. No printouts.**

- Section 1.1: Sets. The relationships $x \in S$, $S \subseteq T$, and $S = T$. (Element of; subset of; and set equality.) Various ways to denote a set, including what the book calls *roster form* and *set-builder notation*. The empty set $\varnothing$. The specific sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$. The power set $\mathcal{P}(S)$ of a set $S$.

- Section 1.2: Set operations. Intersection and union of two sets, of finitely many sets, and of arbitrary collections of sets. Disjoint sets. The relative complement, or set difference, $R \smallsetminus S$ of a set $S$ in another set $R$. The DeMorgan and distributive laws. The (Cartesian) product $S \times T$ of two sets, or $S_1 \times \cdots \times S_n$ of finitely many sets.

- Section 1.4: Logic and Truth Tables. Statements and truth values. The notation $\sim$, $\vee$, $\wedge$ for "not", "or", and "and". The DeMorgan and distributive laws for these symbols. Truth tables — be able to use them.

- Section 1.5: Quantifiers. The existential ($\exists$) and universal ($\forall$) quantifiers. Know what they mean, how to write and interpret logical statements involving them, how to prove statements that involve them, and how to use such statements in proofs. Negations of quantified statements.

- Section 1.6: Implications. The notation $P \Rightarrow Q$, its truth table, and its equivalence to $(\sim P) \vee Q$. Converse, contrapositive, and negation of an implication. Theorem 1.6.2, that an implication is equivalent to its contrapositive. "If and only if", also written iff or $\Leftrightarrow$.

- Section 2.1: Direct Proof. Direct Proof. Direct Proof. Yes, I said that three times, which probably wasn't enough, so here's another: Direct Proof. Indirect Proofs: by contrapositive (for implications, but to be used sparingly), or by contradiction (more general, but still to be used sparingly). Existence and Uniqueness proofs. Proofs of implication, iff, and TFAE statements.

- Section 2.2: Mathematical Induction. Know when to use it (for proofs of statements like "For all $n \in \mathbb{N}, \ldots$," and even then, only sometimes). Know *how* to use it. The base case (or anchor step), the inductive hypothesis (or induction hypothesis), and the inductive step (or induction step). Starting points other than $n = 1$ (as in Example 2.2.5).

**Some things you don't need to know**

The following are various results and concepts that you are allowed to use if you wish, but that you will not need; I will design the exam not expecting you to know them.

- Section 1.1: Cardinality of a set. The set $\mathbb{W}$ of whole numbers.
- Section 1.2: Theorems 1.2.6 and 1.2.9 on the cardinality of product sets. Tree diagrams.
- Section 1.3: This entire section (on partitions).
- Section 1.5: The Archimedian property. The additive identity. The multiplicative identity. Theorem 1.5.2 (the DeMorgan laws for infinite collections of sets).
- Section 1.6: The language "only if" by itself. The language of "necessary" or "sufficient".
- Section 2.2: Strong induction (2.2.7), although you *will* need to know this for future exams. And again, like everything above, you *may* use it if you wish.

**Tips**

- Know all the relevant definitions. That means three things:
  - First, you should know the official definition (not necessarily verbatim, but close enough).
  - Second, you should have a decent intuition for what the concept means informally.
  - Third, you should also be able to use that concept *and* the rigorous definition accurately in a proof.
- Know the theorems from Chapter 1 in the same triple fashion. (I'm not saying this about the theorems from Chapter 2, because most of the "Theorems" in Chapter 2 are really just examples, from our perspective.)
- Given a complicated statement like "For all $n \in \mathbb{Z}$, there is some $x \in \mathbb{R}$ such that $S_n \subseteq [x, \infty)$," be able to break it down into its component pieces, in the appropriate order. Then, to prove it by direct proof, know *exactly* what you need to write to structure that portion of the proof. For example, proving "For all $n \in \mathbb{Z}$, _____" should begin "Given $n \in \mathbb{Z}, \ldots$." Similarly proving "there is some $x \in \mathbb{R}$ s.t. ..." should begin "Let $x = \boxed{\phantom{xx}} \in \mathbb{R}$."
- To study direct proofs, do three main things: learn the definitions (and the theorems from Chapter 1) inside out. Know how to prove all the different little things like $S = T$ or $P \Leftrightarrow Q$. And practice a lot.
- Also practice the other kinds of proofs, too, especially mathematical induction.
- I'll allow you to bring one sheet of handwritten notes to the exam: standard size (8.5x11" or so), and you may use both sides of the sheet. (It may **not** be a printout. You must **hand-write it yourself** on paper with a pen or pencil.) Take some real time to prepare this "cheat sheet." Doing that preparation, especially writing things out yourself on paper, is itself a great study technique.