What you need to know for Exam 1

You should know everything from the beginning of the course up to (and including) Section 13.3. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- **12.1:** Coordinates. \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \) and \( xyz \)-coordinates. The coordinate planes in \( \mathbb{R}^3 \). The distance formula in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \). Equations for spheres.

- **12.2:** Vectors. Addition and scalar multiplication: geometrically (e.g. parallelogram law) and algebraically (e.g. adding coordinates). The vector \( \overrightarrow{AB} \). Length (a.k.a. magnitude) of a vector. Algebraic properties of vectors. \( \vec{i}, \vec{j}, \) and \( \vec{k} \). The zero vector \( \vec{0} \). Parallel vectors.

- **12.3:** Dot Product. Definition and properties of \( \vec{a} \cdot \vec{b} \). Angle \( \theta \) satisfies \( \vec{a} \cdot \vec{b} = \| \vec{a} \| \| \vec{b} \| \cos \theta \); orthogonal iff \( \vec{a} \cdot \vec{b} = 0 \). (Vector) projection \( \text{proj}_{\vec{a}} \vec{b} \).

- **12.4:** Cross Product. (**Only in** \( \mathbb{R}^3 \).) \( 2 \times 2 \) and \( 3 \times 3 \) determinants; definition of cross product. \( \| \vec{a} \times \vec{b} \| = \| \vec{a} \| \| \vec{b} \| \sin \theta \); parallel iff \( \vec{a} \times \vec{b} = \vec{0} \). \( \| \vec{a} \times \vec{b} \| \) is area of parallelogram. Algebraic properties of cross product.

- **12.5:** Lines and Planes. Parametric and symmetric equations for a line; vector parallel to a line. Line segment from \( \vec{r}_0 \) to \( \vec{r}_1 \). Planes; normal vector to a plane.

- **12.6:** Quadric Surfaces. Cylinders (parabolic, circular, elliptic, hyperbolic, or whatever). Be able to figure out the graphs of quadric surfaces of the two simplified equation types listed on the very last line of page 851. (Use “traces” and/or the table on page 854 if they help you.) Practice sketching the various kinds of surfaces.

- **13.1:** Vector-valued Functions. \( \vec{r}(t) = (f(t), g(t), h(t)) \) and its meaning as a parametric curve in space. Limit of a vector-valued function. Space curves (and plane curves).

- **13.2:** Derivatives of vector-valued functions. \( \vec{r}'(t) \): definition (using limits) and properties (vector versions of product rule, chain rule, etc.). Idea as tangent to curve, with length \( \| \vec{r}'(t) \| \) being the speed.

- **13.3:** Arclength. The arclength formula. Know it, and be prepared to compute the resulting integral if I ask you to do so.
Some Things You Don’t Need to Know

- Throughout: proofs of all theorems.
- 12.3: Direction angles and direction cosines.
- 12.3: Scalar projection (a.k.a. component of \( \vec{b} \) along \( \vec{a} \)) \( \text{comp}_{\vec{a}} \vec{b} \).
- 12.4: Triple Product. Torque.
- 12.5: Distance from a point to a plane (formula 9, page 847).
- 12.1 and 12.6: completing the square (which is often used to translate a more general quadric surface to center it at the origin).
- 12.6: all the specialized names like “hyperboloid of two sheets” or “hyperbolic paraboloid”.
- 13.1: using computers to draw space curves.
- 13.3: Unit tangent, normal, and binormal vectors. Curvature. Parametrizing a curve with respect to arclength; smoothness; osculating plane.

Tips

- Don’t mix up vectors and scalars; for example, 0 and \( \vec{0} \) are not the same thing. Dot products output scalars, but scalar multiplication and cross products output vectors. Don’t write a “dot” for multiplication unless you actually mean the dot product.

- Be able to compute the equation for a plane given three points on it, or given a line and a point lying in it, or other such determining information. Similarly, be able to compute an equation for a line given whatever appropriate information — for example, given that it is the intersection of two particular planes. Be able to compute angles between planes and/or lines, and decide whether two lines are skew, parallel, or crossing.

- When drawing graphs of surfaces, get the curves that you draw around the outer edges right. (Don’t worry about trying to shade the surface; no amount of shading will help if the outer edges look wrong.) For example, when drawing a parabolic cylinder, sketch two identical copies of the parabola at both ends, and then connect them by straight lines parallel to the appropriate axis. So if the cylinder runs vertically, make the lines on the side truly vertical (parallel to the \( z \)-axis); or if it runs along the \( x \)-axis, make the lines along the sides parallel to the \( x \)-axis that you drew. It often helps to draw a practice sketch first, and then separately draw the real sketch once you have a better idea of how it’s actually going to look. Make your sketch big.

- For all the definitions (e.g. dot product, cross product, derivative of a vector-valued function), you should have an intuitive idea of what it means and you should know the official definition (e.g. the formulas for the various products, the limit definition of the derivative). In addition, you should know how to compute with all those sorts of objects and formulas.