Practice Problems for the Final Exam
(Warning: these problems only cover the material since Midterm 3. The actual final exam will be comprehensive.)

1. Let \( \mathbf{F}(x,y,z) = \langle y, z^2, x \rangle \). Compute \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \) and \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \), where \( C_1 \) is the helix \( x = 4 \cos t, y = 4 \sin t, z = t \) from \((4,0,0)\) to \((4,0,2\pi)\), and \( C_2 \) is the line segment also running from \((4,0,0)\) to \((4,0,2\pi)\).

2. Evaluate the following line integrals.
   (a). \( \int_C z \, dx - y \, dy - x \, dz \), where \( C \) is parametrized by \( \mathbf{r}(t) = 5 \mathbf{i} - \sin t \mathbf{j} - \cos t \mathbf{k} \), for \( 0 \leq t \leq \pi/4 \).
   (b). \( \int_C (y + 2xe^y) \, dx + (x + x^2e^y) \, dy \), where \( C \) is parametrized by \( \mathbf{r}(t) = \sqrt{t} \mathbf{i} + \ln t \mathbf{j} \) for \( 1 \leq t \leq 4 \).
   (c). \( \int_C y \cos x \, dx + x \sin y \, dy \), where \( C \) is the boundary of the triangle with vertices \((0,0)\), \((\pi/2,0)\), and \((0,\pi/2)\), oriented clockwise.

3. Find the work done by the force \( \mathbf{F}(t) = \langle x^2y, z, 2x - y \rangle \) moving an object along the straight line segment from \((1,1,1)\) to \((2,-3,3)\).

4. Let \( \mathbf{F} \) and \( \mathbf{G} \) be the vector fields \( \mathbf{F}(x,y,z) = (x \sin y + z) \mathbf{i} + xy \mathbf{j} + xz \mathbf{k} \) and \( \mathbf{G}(x,y) = (e^x + \cos(xy) - xy \sin(xy)) \mathbf{i} + (2y - x^2 \sin(xy)) \mathbf{j} \).
   (a). Compute \( \text{curl} \mathbf{F} \) and \( \text{div} \mathbf{F} \).
   (b). Is \( \mathbf{F} \) conservative (on \( \mathbb{R}^3 \))? If so, express it as a gradient; if not, explain why not.
   (c). Is \( \mathbf{G} \) conservative (on \( \mathbb{R}^2 \))? If so, express it as a gradient; if not, explain why not.

5. Let \( C \) be the circle \( x^2 + y^2 = 4 \), oriented counterclockwise.
   Compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x,y) = \langle xy^2 - y^3, x^3 + x^2y \rangle \).

6. Determine whether each of the following vector fields is conservative. If it is, find a potential function for it.
   (a). \( \mathbf{F}(x,y) = \langle x^2 + 2x \sin y, 5 + x^2 \cos y \rangle \)  
   (b). \( \mathbf{G}(x,y,z) = \langle y + z, 2z, x + y \rangle \)  
   (c). \( \mathbf{H}(x,y,z) = 3x^2 \mathbf{i} + \frac{z}{y} \mathbf{j} + \ln y \mathbf{k} \)

7. Let \( \mathbf{F}(x,y) = \left\langle \frac{xy^2}{1 + x^2 y}, \frac{x^2 y}{1 + x^2} \right\rangle \).
   (a). Show that \( \mathbf{F} \) is conservative, and find an associated potential function.
   (b). Compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the curve parametrized by \( \mathbf{r}(t) = \langle \cos(\pi t^3), \sqrt{1 + t^3} \rangle \), for \( 0 \leq t \leq 2 \).
8. Compute the divergence and the curl of the following vector fields. Decide whether each is the gradient of something, or the curl of something, or both, or neither.

(a). \( \vec{F}(x, y, z) = (x + y, y + z, x + z) \)

(b). \( \vec{G}(x, y, z) = x^2 \vec{i} - xe^y \vec{j} + 2xyz \vec{k} \)

(c). \( \vec{H}(x, y, z) = (x - yz, y - xz, xy - 2z) \)

9. Compute \( \int_C \sqrt{1 + x^3} \, dx + 2xy \, dy \), where \( C \) is the triangle with vertices \((0, 0), (1, 0), \) and \((1, 3)\), oriented counterclockwise.

10. Evaluate the following surface integrals.

(a). \( \int\int_S \langle x, -1, 2x^2 \rangle \cdot d\vec{S} \), where \( S \) is the part of the paraboloid \( z = x^2 + y^2 \) above the region in the \( xy \)-plane bounded by \( x = 1 - y^2 \) and \( x = y^2 - 1 \), oriented upward.

(b). \( \int\int_S \langle x, 2x + z, y - z \rangle \cdot d\vec{S} \), where \( S \) is the portion of the cone \( z = \sqrt{x^2 + y^2} \) beneath the plane \( z = 2 \), oriented downward.

(c). \( \int\int_S \langle x^2, xy, -2xz \rangle \cdot d\vec{S} \), where \( S \) is the tetrahedron with vertices \((0, 0, 0), (1, 0, 0), (0, 1, 0), \) and \((0, 0, 1)\), oriented outward.

11. Let \( \vec{F}(x, y, z) = \langle x^2 + y^2, \cos(xz), yz + \sec(x^3) \rangle \). Let \( S \) be the surface of the rectangular box \([0, 1] \times [0, 2] \times [1, 3] \), oriented outward. Compute \( \int\int_S \vec{F} \cdot d\vec{S} \).

12. Let \( \vec{G}(x, y, z) = \langle x, y, z^2 \rangle \). Let \( S \) be the portion of the sphere \( x^2 + y^2 + z^2 = 4 \) in the first octant, oriented upward. Compute \( \int\int_S \vec{G} \cdot d\vec{S} \).