Math 211, Section 01, Fall 2019

**Practice Problems for Midterm Exam 3**
(A little harder than, and about three times as long as, the real exam)

1. For the following integrals, sketch the region of integration, and then reverse the order of integration.
   \[
   \int_0^1 \int_{1-x}^{1-x^2} f(x, y) \, dy \, dx \quad \text{and} \quad \int_0^{\ln 3} \int_{e^y}^3 g(x, y) \, dx \, dy
   \]

2. Compute \( \iint_D 2x + y \, dA \), where \( D \) is the region in the plane bounded by the line \( y = 3 - 2x \) and the hyperbola \( xy = 1 \).

3. Compute \( \iint_D x^2 y \, dA \), where \( D \) is the region in the first quadrant inside the circle \( x^2 + y^2 = 4 \) and outside the circle \( x^2 + y^2 = x \).

4. Compute \( \iint_D 4 + x^2 \, dA \), where \( D \) is the region in the plane bounded by the parabolas \( y = x^2 + 1 \) and \( y = 3 - x^2 \).

5. Compute \( \iint_D x^3 + xy^2 \, dA \), where \( D \) is the region in the first quadrant bounded by the circle \( x^2 + y^2 = 1 \) and the lines \( y = x \) and \( y = \sqrt{3}x \).

6. Compute \( \iint_D x^2 + y \, dA \), where \( D \) is the region in the plane bounded above by the curves \( y = x^2 \) and \( y = 6 - x \) and below by the \( x \)-axis.

7. Let \( D \) be the region in the plane that lies inside the circle \( x^2 + y^2 = 6y \) but outside the circle \( x^2 + y^2 = 9 \). Compute the area of \( D \).

8. Compute \( \int_0^1 \int_{x^{1/3}}^1 \cos(\pi y^4/2) \, dy \, dx \).

9. Find the volume of the solid that lies between the paraboloids \( z = x^2 + y^2 \) and \( z = 36 - 3x^2 - 3y^2 \).

10. Find the volume of the solid bounded by the surface \( z = e^x \) and the planes \( x = y, x = 1, y = 0, \) and \( z = 0 \).

11. Let \( E \) be the tetrahedron with vertices \((0, 0, 0), (1, 0, 0), (0, 2, 0), \) and \((0, 0, 4)\), and with constant density \( 1 \). Find the mass and center of mass of \( E \).

12. Let \( E \) be the solid region below the plane \( z = 10 - x \), above the plane \( z = x + y \), and between the surfaces \( x = y^2 \) and \( y = x - 2 \). Compute \( \iiint_E y \, dV \).

13. Let \( E \) be the solid region between the parabolic cylinder \( y = 2z^2 \) and the plane \( y = -2z \), bounded in front by the plane \( x = 4 \) and in back by the plane \( x + z = y \). Compute \( \iiint_E z^2 \, dV \).
14. Let \( E \) be the solid lying inside the sphere \( x^2 + y^2 + z^2 = 4 \) and below the cone \( z = \sqrt{x^2 + y^2} \). Compute \( \iiint_E z^2 \, dV \).

15. Let \( E \) be the portion of the solid in the first octant bounded by the \( xz \)-plane, the \( yz \)-plane, the plane \( z = 4 \), and the paraboloid \( z = x^2 + y^2 \). Compute \( \iiint_E x \, dV \).

16. Let \( E \) be the solid region between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 2 \), and inside the (upper half of the double) cone \( z = \sqrt{x^2 + y^2} \). The density of \( E \) at \( (x, y, z) \) is \( z^2 \). Compute the mass of \( E \).

17. Let \( a > 0 \). Find the volume of the solid inside both the sphere \( x^2 + y^2 + z^2 = a^2 \) and the cylinder \( x^2 + y^2 = ax \), and in the first octant.

18. Let \( E \) be the solid bounded by the surfaces \( y = 1 - x^2 \), \( y = 0 \), \( z = y \), and \( z = 1 \). Suppose that the density of the solid is \( \rho(x, y, z) = 2(1 + x^4) \). Compute the mass of \( E \).

19. Compute \( \int_{-3}^{3} \int_{0}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy \).

20. Let \( E \) be the solid between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 9 \), below the cone \( z = \sqrt{x^2 + y^2} \), and above the cone \( z = -\sqrt{3x^2 + 3y^2} \). Compute \( \iiint_E x^2 \, dV \).

21. Let \( E \) be the solid between the paraboloid \( z = x^2 + y^2 \) and the cone \( z = 2\sqrt{x^2 + y^2} \) in the first octant. Compute \( \iiint_E y^2 \, z \, dV \).