1. Each of the following limits converges. Compute them.
\[
\lim_{(x,y) \to (0,0)} \frac{e^x + x \sin y}{\ln(1 + x) + \cos^2(xy)} = \lim_{(x,y) \to (1,2)} \frac{x^4 - 3y^2}{x^2 + y^2} = \lim_{(x,y) \to (0,0)} \frac{x^4 + 2x^2 + x^2y^2 + 2y^2}{x^2 + y^2}
\]

2. Prove that each of the following limits diverges.
\[
\lim_{(x,y) \to (0,0)} \frac{2x^2 - y^2}{x^2 + 2y^2} = \lim_{(x,y) \to (0,0)} \frac{x^2y + y^3 + xy}{x^2 + y^2} = \lim_{(x,y) \to (0,0)} \frac{4x^2y^3 + 3xy^4}{x^8 + 5y^4}
\]

3. Let \( f(x, y) = \begin{cases} 
\frac{x^3 + 2y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0).
\end{cases} \)
   
   (a). Compute \( f_x(0, 0) \) and \( f_y(0, 0) \).
   
   (b). Compute \( D_\vec{u} f(0, 0) \), where \( \vec{u} \) is the unit vector in the direction of \( \vec{v} = (1, 1) \).
   
   (c). Is \( f \) differentiable at \( (0, 0) \)? Why or why not?

4. State, but do not prove, the \( \varepsilon-\delta \) definition of each of the following limit statements.
\[
\lim_{(x,y) \to (4, -5)} H(x, y) = -3 \\
\lim_{(x,y) \to (3,2)} \sqrt{2x^2 - y} = 4 \\
\lim_{(x,y,z) \to (0,-2,1)} Q(x, y, z) = R
\]

5. Let \( f(x, y) = \frac{20 + \sin(x + 4y)}{10 + 3y} \).
   
   (a). Find an equation for the tangent plane to the surface \( z = f(x, y) \) at the point where \( (x, y) = (0, 0) \).
   
   (b). Use your tangent plane to approximate the value of \( f(-0.3, 0.1) \).

6. Let \( g(x, y) = x^3y - 2x^2 + \frac{4y}{x} \).
   
   (a). Is \( g \) differentiable at \( (0, 3) \)? Why or why not?
   
   (b). Is \( g \) differentiable at \( (2, 3) \)? Why or why not?
   
   (c). Compute \( D_\vec{u} f(2, 3) \), where \( \vec{u} \) is the unit vector in the direction of \( \vec{v} = (3, -4) \).
   
   (d). At \( (2, 3) \), in what direction does \( g \) increase the fastest?
   
   (e). What is the rate of increase of \( g \) in the direction from part (d)?

7. Suppose \( f(x, y), g(s, t), \) and \( h(s, t) \) are differentiable functions such that:
\[
\begin{align*}
&f_x(0, 0) = -4, \quad f_y(0, 0) = 7, \quad f_x(3, -2) = 5, \quad f_y(3, -2) = -2, \\
&f(0, 0) = 2, \quad f(3, -2) = 9, \quad g_s(0, 0) = -1, \quad g_t(0, 0) = 4, \\
&h_s(0, 0) = 2, \quad h_t(0, 0) = 3, \quad g(0, 0) = 3, \quad h(0, 0) = -2.
\end{align*}
\]

Let \( F(s, t) = f(g(s, t), h(s, t)) \). Compute \( F_s(0, 0) \) and \( F_t(0, 0) \).
8. Suppose $H(x, y, z)$ is a differentiable function such that $\nabla H(3, 2, 1) = (-4, 6, 5)$. Let $f(t) = H(t^2 - 1, 8 - 3t, t - 1)$.
   (a) Compute $f'(2)$.
   (b) At the point $(3, 2, 1)$, in which direction does $H(x, y, z)$ increase the fastest? How fast does $H$ increase when we move in that direction?

9. Find an equation for the tangent plane to $z = \sqrt{xy} + 3y$ at the point $(1, 1, 2)$.

10. Find an equation for the tangent plane to $x^3 - \cos(xz) + xy^3 - yz = 0$ at the point $(0, 1, -1)$.

11. Find any and all point(s) on the ellipsoid $4x^2 + y^2 + 4z^2 = 36$ at which the tangent plane is parallel to the plane $x + y - 2z = 3$.

12. Find and classify (as local minimum, local maximum, or saddle point) all critical points of the function $f(x, y) = 4xy - x^4 - y^4$.

13. Find and classify (as local minimum, local maximum, or saddle point) all critical points of the function $g(x, y) = 8x^3 - 12xy + y^3 - 7$.

14. Find the point(s) on the hyperboloid $x^2 + y^2 - 4z^2 = 10$ closest to the point $(4, 2, 0)$.

15. Find the maximum and minimum values of the function $g(x, y) = 4xy + y^2$ on the ellipse $9x^2 + y^2 = 45$.

16. Find the maximum and minimum values of the function $f(x, y, z) = xyz$ on the sphere $x^2 + y^2 + z^2 = 3$.

17. Find the maximum and minimum values of the function $g(x, y) = 3x^2 - 4y + y^2 + 2$ on the disk $x^2 + y^2 \leq 9$. 