Math 211, Section 01, Fall 2019

Fall 2018 Math 211-01,05 Final Exam

Instructions: Do all twelve numbered problems. If you wish, you may also attempt the three optional bonus questions. Show all work, including scratch work. Little or no credit may be awarded, even when your answer is correct, if you fail to follow instructions for a problem or fail to justify your answer. If your answer for a given problem is a sum of fractions with different denominators, you may leave it that way. Otherwise, simplify your answers whenever possible. If you need more space, use the back of any page. If you have time, check your answers.

WRITE LEGIBLY. NO CALCULATORS.

1. (12 points) Find an equation for the line of intersection of the planes \( x - 5y - 2z = 3 \) and \( y - z = 2 \).

2. (12 points) Let \( f(x, y) = \sqrt{5 - 2xy} \) Find an equation of the tangent plane to the surface \( z = f(x, y) \) at the point \((2, 1, 1)\), and then use it to estimate \( f(1.9, 1.1) \).

3. (16 points) Let \( f(x, y) = \begin{cases} 6x^3 - 5xy \quad & \text{if } (x, y) \neq (0, 0), \\ x^2 + 2y^2 \quad & \text{if } (x, y) = (0, 0). \end{cases} \)

3a. Compute \( f_x(0, 0) \) and \( f_y(0, 0) \).

3b. Prove that \( f \) is not continuous at \((0, 0)\).

4. (20 points) Find and classify (as local minimum, local maximum, or saddle point) every critical point of the function \( f(x, y) = 2x^3 - y^2 + 6xy \).

5. (20 points) Find the maximum and minimum values of the function \( f(x, y) = x^2y \) subject to the constraint \( x^2 + y^2 = 9 \).

6. (20 points) Let \( E \) be the solid bounded by the surfaces \( y = \sqrt{x}, x = 2y, z = 4, \) and \( x + z = 4 \). Suppose the density of \( E \) is given by \( \rho(x, y, z) = y \). Compute the mass of \( E \).

7. (15 points) Let \( \vec{G}(x, y, z) = (x^2 - 5yz, xy + z, y^2 - 3xz) \).

7a. Compute \( \text{curl } \vec{G} \) and \( \text{div } \vec{G} \).

7b. Is \( \vec{G} \) equal to the gradient of anything? Why or why not?

[If so, you do not need to find what it is the gradient of.]

7c. Is \( \vec{G} \) equal to the curl of anything? Why or why not?

[If so, you do not need to find what it is the curl of.]

8. (15 points) Let \( C \) be the straight line segment from \((2, 0, -3)\) to \((1, 2, 0)\). Compute \( \int_C \vec{F} \cdot d\vec{r} \), where \( \vec{F}(x, y, z) = (z, 3y, xy) \).

9. (18 points) Let \( S \) denote the surface bounding the solid in the first octant enclosed by the sphere \( x^2 + y^2 + z^2 = 9 \), the cone \( z = \sqrt{x^2 + y^2} \), the \( xz \)-plane, and the \( yz \)-plane. If we orient \( S \) outward and set \( \vec{F}(x, y, z) = xi - yj + z^2k \), compute \( \iint_S \vec{F} \cdot d\vec{S} \).
10. **(15 points)** Let \( \vec{F}(x, y) = \langle x - \cos(2y), y^3 + 2x \sin(2y) \rangle \).

10a. Show that \( \vec{F} \) is conservative by finding a potential function for \( \vec{F} \).

10b. Let \( C \) be the curve parametrized by \( \vec{r}(t) = \langle \sqrt{t^2 + 9}, e^{t^2 - 4t} \rangle \) for \( 0 \leq t \leq 4 \).

Compute \( \int_C \vec{F} \cdot d\vec{r} \).

11. **(15 points)** Let \( C \) be the curve in the plane that starts at the origin, goes straight up the \( y \)-axis to the point \((0, 2)\), then proceeds **counterclockwise** around the circle \( x^2 + y^2 = 4 \) to the point \((-2, 0)\), and finally goes straight to the right along the \( x \)-axis back to the origin, as in this picture:

![Image](image.png)

Compute \( \int_C (y^3 + 5x^4 y^2) \, dx + (5xy^2 + 2x^5 y) \, dy \).

12. **(22 points)** Let \( \vec{F}(x, y, z) = \langle y^2, -xy, z^2 \rangle \), and let \( S \) be the part of the paraboloid \( z = 3x^2 + 3y^2 \) in the first octant and below the plane \( z = 3 \), oriented downward.

Compute \( \iint_S \vec{F} \cdot d\vec{S} \).