Math 211, Section 05, Fall 2018

Midterm Exam 3, Friday, November 30, 2018

Instructions: Do all five numbered problems. If you wish, you may also attempt the two optional bonus questions. Show all work, including scratch work. Little or no credit may be awarded, even when your answer is correct, if you fail to follow instructions for a problem or fail to justify your answer. If your answer for a given problem is a sum of fractions with different denominators, you may leave it that way. Otherwise, simplify your answers whenever possible. If you need more space, use the back of any page. If you have time, check your answers.

WRITE LEGIBLY. NO CALCULATORS.

1. (15 points) Consider the integral \( \int_{-2}^{0} \int_{x^2}^{4} f(x, y) \, dy \, dx \)
   (a). Sketch the region of integration.
   (b). Rewrite the integral with the order of integration reversed.

2. (18 points) Let \( D \) be the region in the plane
   • inside the circle \( x^2 + y^2 = 2y \) (which is not centered at the origin), and
   • below the line \( y = x \).
   Compute \( \iint_{D} 2xy \, dA \).

3. (22 points) Let \( E \) be the solid that is
   • below (i.e., outside) the cone \( z = \sqrt{x^2 + y^2} \),
   • in front of the \( yz \)-plane, and
   • inside the sphere \( x^2 + y^2 + z^2 = 4 \).
   Compute the mass of \( E \), if the density of \( E \) at the point \( (x, y, z) \) is \( x \).

4. (20 points) Let \( E \) be the solid enclosed by the paraboloid \( z = 1 - x^2 - y^2 \) and the coordinate planes in the first octant. Compute \( \iiint_{E} x^3 \, dV \).

5. (25 points) Let \( E \) be the solid bounded by the surfaces \( z = \sqrt{x}, \, z = x, \, y = 0, \) and \( y + 2z = 2 \). Suppose the density of \( E \) is given by \( \rho(x, y, z) = x \).
   Take my word for it that the mass of \( E \) is \( 1/20 \). Compute the \( z \)-coordinate of the center of mass of \( E \).

OPTIONAL BONUS A. (2 points) Let \( E \) be the solid in the first octant bounded by the coordinate planes and the two circular cylinders \( x^2 + z^2 = 1 \) and \( y^2 + z^2 = 1 \). Compute \( \iiint_{E} x^2 z^2 \, dV \).

OPTIONAL BONUS B. (1 point) Name the capitals of two of the following three countries: Sudan, Indonesia, and Honduras.