Midterm Exam 2, Wednesday, October 31, 2018

Instructions: Do all seven numbered problems. If you wish, you may also attempt the two optional bonus questions. Show all work, including scratch work. Little or no credit may be awarded, even when your answer is correct, if you fail to follow instructions for a problem or fail to justify your answer. Simplify your answers whenever possible, and draw your graphs large and clearly labelled. If you need more space, use the back of any page. If you have time, check your answers.

WRITE LEGIBLY. NO CALCULATORS.

1. (12 points) Show that \( \lim_{{(x,y) \to (0,0)}} \frac{5xy - 2y^3}{2x^2 + y^2} \) diverges.

2. (8 points) State the formal definition (i.e., the \( \varepsilon-\delta \) definition) of \( \lim_{{(x,y) \to (-2,5)}} x^7 + y^3 = -3. \)

3. (6 points) Let \( h(x, y) = x^2 - 3xe^y. \) In which direction does \( h \) increase the fastest at the point \((1,0)\)?

4. (20 points, 2 parts) Let \( f(x, y) = \begin{cases} \frac{3y^3 - x^2y}{x^2 + y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0). \end{cases} \)

   (4a). Compute \( f_x(0,0) \) and \( f_y(0,0). \)

   (4b). Compute \( D_u f(0,0), \) where \( u = \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle. \)

5. (12 points) Let \( h(x, y) \) be a differentiable function such that
   \[ \nabla h(3,-1) = \langle -6,5 \rangle, \quad \nabla h(-2,-3) = \langle 7,-2 \rangle, \quad \text{and} \quad \nabla h(1,-1) = \langle -2,3 \rangle. \]

   Define \( f(s, t) = h(s + 5t^3, st). \) Compute \( f_s(3,-1). \)

6. (22 points) Find and classify (as local minimum, local maximum, or saddle point) all critical points of the function \( h(x, y) = x^2y + 5x^2 + 10y^2 - 3. \)

7. (20 points) Find the maximum and minimum values of the function \( f(x, y) = x^2y \) on the ellipse \( x^2 + 2y^2 = 6. \)

OPTIONAL BONUS A. (2 points) Prove that the function

\[
f(x, y) = \begin{cases} \frac{6x^2 - 3x^2y^2 + 4y^2}{3x^2 + 2y^2} & \text{if } (x, y) \neq (0,0) \\ 2 & \text{if } (x, y) = (0,0) \end{cases}
\]

is differentiable at \((0,0).\)

OPTIONAL BONUS B. (1 point) A month ago, on September 28, 2018, a major earthquake struck near the city of Palu, killing hundreds. In what country is Palu located?