Homework #3

Due **Friday, September 13** in Gradescope by **11:59 pm ET**

- **READ** the two worked-out "LIDS" derivative proofs in this handout
- **WRITE AND SUBMIT** solutions to the 21 assigned problems in this handout

NOTE: Show your work! The two examples below are good models for the two proof problems (9 and 10). For the other problems, look to the worked-out examples on prior HW's for good models for how to structure your solutions.

Example 1: **PROVE** that $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$

Proof: Let $y = \arctan x$ (We're hoping to solve for $\dfrac{dy}{dx}$ here.)

Invert to get $\tan y = x$

Differentiate to get $\dfrac{d}{dx}(\tan y) = \dfrac{d}{dx}(x)$

So: $\sec^2 y \cdot \dfrac{dy}{dx} = 1$

Solve to get $\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+\tan^2 y} = \dfrac{1}{1+(\tan y)^2} = \dfrac{1}{1+x^2}$ QED

Note: "QED" is an abbreviation for the Latin "Quod Erat Demonstrandum," which literally translates to "which was to be shown." In other words, at the end of a proof, QED means, "Yes, that's what we were trying to do! So we're done!"

Example 2: **PROVE** that $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$

Proof: Let $y = \arcsin x$ (We're hoping to solve for $\dfrac{dy}{dx}$ here.)

Invert to get $\sin y = x$

Differentiate to get $\dfrac{d}{dx}(\sin y) = \dfrac{d}{dx}(x)$

So: $\cos y \cdot \dfrac{dy}{dx} = 1$

Solve to get $\dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\sqrt{1-\sin^2 y}} = \dfrac{1}{\sqrt{1-(\sin y)^2}} = \dfrac{1}{\sqrt{1-x^2}}$ QED

Next, complete the following HW problems

found on the next page

Assigned Problems for HW 3

Exercises 1–4: Differentiate the following functions. Simplify.

1. $f(x) = \tan^{-1}(x^2)$
2. $f(x) = \left(\tan^{-1}(x)\right)^2$
3. $y = x\sin^{-1} x + \sqrt{1-x^2}$
4. $f(x) = \ln\left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)$

5. Find the value of the expression $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

6. Simplify the expression $\sin(\tan^{-1} x)$

7. Compute the Second Derivative of $f(x) = \arctan(2x)$

8. Compute the Second Derivative of $f(x) = \arcsin(6x)$

9. **Prove** that $\frac{d}{dx}\sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$ using a "LIDS" proof.

10. **Prove** that $\frac{d}{dx}\tan^{-1}(5x) = \frac{5}{1+25x^2}$ using a "LIDS" proof.

11. Use our Integration Methods to **Justify** that $\int \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$
You may **not** use an "a-rule", but instead work it out using algebra, substitution, and the fact that $\int \frac{du}{1+u^2} = \arctan u + C$.

Exercises 12–21: Evaluate each of the following Integrals. Simplify. Justify.

12. $\int \frac{x^2}{x^2+1}\,dx$
13. $\int \frac{x+1}{x^2+1}\,dx$
14. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$
15. $\int_0^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$
16. $\int \frac{1}{\sqrt{1-x^2}\cdot\sin^{-1} x}\,dx$
17. $\int_1^3 \frac{1}{\sqrt{x}\,(1+x)}\,dx$
18. $\int_0^{\ln 3} \frac{e^x}{1+e^x}\,dx$
19. $\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}}\,dx$
20. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$
21. $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2}\,dx$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm
Thursday: 1:30-3:00 pm
Friday: 2:00–3:00 pm
(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

- My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.
- **NO LATE HOMEWORK!** unless illness or emergency occurs.