Homework #2
Due **Wednesday, September 11** in Gradescope by **11:59 pm ET**

- **READ** the five worked-out examples in this handout

- **WRITE AND SUBMIT** solutions to the 20 assigned problems in this handout

NOTE: Show your work! Use the solutions to the eight worked examples here as guides for how much work to show on these sorts of problems.

Think about the graph of $y = \ln x$. We know that $\boxed{\lim_{x \to 0^+} \ln x = \lim_{x \to 0^+} \ln x \overset{0^+}{=} -\infty}$. Learn this!

Example 1: $\lim_{x \to 5^+} \ln(x - 5) = \lim_{x \to 5^+} \ln(x - 5) \overset{5^+}{\underset{}{\nearrow}} {}^{0^+} = -\infty$

Example 2: $\lim_{x \to 8^-} \ln|x - 8| = \lim_{x \to 8^-} \ln|x - 8| \overset{0^+ / 0^-}{=} -\infty$

Example 3: $\displaystyle \int \frac{(3 - \sqrt{x})(1 + 2\sqrt{x})}{x^2}\, dx = \int \frac{3 + 6\sqrt{x} - \sqrt{x} - 2x}{x^2}\, dx = \int \frac{3 + 5\sqrt{x} - 2x}{x^2}\, dx$

$\displaystyle = \int \frac{3}{x^2} + \frac{5\sqrt{x}}{x^2} - \frac{2x}{x^2}\, dx = \int \frac{3}{x^2} + \frac{5}{x^{3/2}} - \frac{2}{x}\, dx \overset{\text{prep}}{=} \int 3x^{-2} + 5x^{-3/2} - \frac{2}{x}\, dx$

$\displaystyle = -3x^{-1} + 5(-2)x^{-1/2} - 2\ln|x| + C = \boxed{-\frac{3}{x} - \frac{10}{\sqrt{x}} - 2\ln|x| + C}$

Example 4: $\displaystyle \int_{\ln 3}^{\ln 8} \frac{e^x}{\sqrt{1 + e^x}}\, dx$ $\boxed{\begin{aligned} u &= 1 + e^x \\ du &= e^x\, dx \end{aligned}}$ and $\boxed{\begin{aligned} x = \ln 3 &\Rightarrow u = 1 + e^{\ln 3} = 1 + 3 = 4 \\ x = \ln 8 &\Rightarrow u = 1 + e^{\ln 8} = 1 + 8 = 9 \end{aligned}}$

$\displaystyle = \int_4^9 \frac{1}{\sqrt{u}}\, du = \int_4^9 u^{-\frac{1}{2}}\, du = 2\sqrt{u}\Big|_4^9 = 2\sqrt{9}^{\,3} - 2\sqrt{4}^{\,2} = 6 - 4 = \boxed{2}$

Example 5: $\displaystyle \int_1^2 \frac{1}{3 - 5x}\, dx$ $\boxed{\begin{aligned} u &= 3 - 5x \\ du &= -5\, dx \\ -\tfrac{1}{5}du &= dx \end{aligned}}$ and $\boxed{\begin{aligned} x = 1 &\implies u = 3 - 5 = -2 \\ x = 2 &\implies u = 3 - 10 = -7 \end{aligned}}$

$\displaystyle = \frac{1}{5}\int_{-2}^{-7} \frac{1}{u}\, du = -\frac{1}{5}\ln|u|\Big|_{-2}^{-7} = -\frac{1}{5}\left(\ln|-7| - \ln|-2|\right) = \boxed{-\frac{1}{5}\ln\left(\frac{7}{2}\right)}$

Next, complete the following HW problems
found on the next page

Assigned Problems for HW 2

Exercises 1–8: Differentiate the following functions. Simplify.

1. $f(x) = e^5$
2. $f(x) = e^x + x^e$
3. $y = \dfrac{1 - e^{2x}}{1 + e^{2x}}$
4. $f(x) = e^{\sin(2x)} + \sin\left(e^{2x}\right)$

5. $y = e^{\sqrt{x}}$
6. $y = x^2 e^{-1/x}$
7. $y = \ln\left(1 + e^{3x}\right)$
8. $f(x) = \ln\left(\dfrac{1}{x}\right) + \dfrac{1}{\ln x}$

Exercise 9: Express the quantity as a single logarithm. Simplify.

$$\frac{1}{3}\ln[(x+2)^3] + \frac{1}{2}\left[\ln x - \ln[(x^2 + 3x + 2)^2]\right]$$

Exercises 10–11: Solve each of the following equations for x:

10. $e^{7-4x} = 6$
11. $\ln(3x - 10) = 2$

Exercises 12–13: Evaluate each of the following Limits:

12. $\lim\limits_{x \to 2^-} \ln|x - 2|$
13. $\lim\limits_{x \to 3^+} \ln(x^2 - 9)$

Exercises 14–20: Evaluate each of the following Integrals. Simplify. Justify.

14. $\displaystyle\int e^x + x^e \, dx$
15. $\displaystyle\int_0^{\ln 4} \frac{1}{e^{2x}} \, dx$
16. $\displaystyle\int \frac{(1 + e^x)^2}{e^x} \, dx$

17. $\displaystyle\int \left(e^x + e^{-x}\right)^2 \, dx$
18. $\displaystyle\int \frac{e^x}{1 + e^x} \, dx$
19. $\displaystyle\int_2^3 \frac{1}{5 - 4x} \, dx$

20. $\displaystyle\int_e^{e^3} \frac{4}{x(\ln x)^2} \, dx$

My Office Hours: SMUD 406
Tuesday: 1:30–3:00 pm
Thursday: 1:30-3:00 pm
Friday: 2:00–3:00 pm
(or by appointment)

Math Fellow evening TA Help Hours start soon; details TBA

● Office Hours are times to drop in to my office, unannounced. All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

● **NO LATE HOMEWORK!** unless illness or emergency occurs.