Homework #9

Due **Wednesday, October 9** in Gradescope by **11:59 pm ET**

- **READ** the three worked-out examples in this handout
- **CONSULT** Section 7.8 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 11 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\displaystyle\int_{-\infty}^{7} \frac{1}{x^2-6x+25}\,dx = \lim_{t\to-\infty}\int_{t}^{7} \frac{1}{x^2-6x+25}\,dx$

$\overset{\text{complete square}}{=} \displaystyle\lim_{t\to-\infty}\int_{t}^{7} \frac{1}{(x-3)^2+16}\,dx$ $\quad \boxed{\begin{array}{ll} u &= x-3 \\ du &= dx \end{array}} \quad \boxed{\begin{array}{l} x=t \Rightarrow u = t-3 \\ x=7 \Rightarrow y = 7-3 = 4 \end{array}}$

$= \displaystyle\lim_{t\to-\infty}\int_{t-3}^{4} \frac{1}{u^2+16}\,du = \lim_{t\to-\infty}\frac{1}{4}\arctan\left(\frac{u}{4}\right)\Big|_{t-3}^{4}$

$= \displaystyle\lim_{t\to-\infty}\frac{1}{4}\left(\arctan\left(\frac{4}{4}\right) - \arctan\left(\frac{t-3}{4}\right)\right) = \lim_{t\to-\infty}\frac{1}{4}\left(\arctan(1) - \arctan\left(\frac{t-3}{4}\right)\right)$ ($\frac{t-3}{4} \to -\infty$)

$= \frac{1}{4}\left(\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right) = \frac{1}{4}\left(\frac{3\pi}{4}\right) = \boxed{\frac{3\pi}{16}}$

Example 2: $\displaystyle\int_{0}^{1} \frac{\ln x}{\sqrt{x}}\,dx = \lim_{s\to0^+}\int_{s}^{1} \frac{\ln x}{\sqrt{x}}\,dx = \lim_{s\to0^+}\int_{s}^{1} (\ln x)x^{-1/2}\,dx$

IBP: $\begin{array}{ll} u = \ln x & dv = x^{-1/2}dx \\ du = \dfrac{1}{x}\,dx & v = 2\sqrt{x} \end{array}$ $\quad = \displaystyle\lim_{s\to0^+} 2\sqrt{x}\ln x\Big|_{s}^{1} - 2\int_{s}^{1}\frac{1}{\sqrt{x}}\,dx$

$= \displaystyle\lim_{s\to0^+} 2\sqrt{x}\ln x\Big|_{s}^{1} - 4\sqrt{x}\Big|_{s}^{1} = \lim_{s\to0^+} 2\ln 1 - 4\sqrt{1} - \left(2\sqrt{s}\ln s - 4\sqrt{s}\right) = 0 - 4 - \lim_{s\to0^+}\sqrt{s}\ln s$

$\overset{0\cdot(-\infty)}{=} -4 - \displaystyle\lim_{s\to0^+}\frac{\ln s}{s^{-1/2}} \overset{\frac{-\infty}{\infty}}{\overset{\text{L'H}}{=}} -4 - \lim_{s\to0^+}\frac{s^{-1}}{-\frac{1}{2}s^{-3/2}} = -4 + \lim_{s\to0^+} 2\sqrt{s} = -4 + 2\sqrt{0} = \boxed{-4}$

Examples Continue on Next Page

Example 3: $\displaystyle\int_0^6 \frac{8}{x^2-4x-12}\,dx = \int_0^6 \frac{8}{(x-6)(x+2)}\,dx = \lim_{t\to 6^-}\int_0^t \frac{8}{(x-6)(x+2)}\,dx$

$\overset{\text{PFD}}{=} \lim_{t\to 6^-}\int_0^t \frac{1}{x-6}-\frac{1}{x+2}\,dx = \lim_{t\to 6^-} \ln|x-6|-\ln|x+2|\Big|_0^t$

$= \lim_{t\to 6^-} \ln|t-6| - \ln|t+2| - (\ln|-6| - \ln 2)$ ($t-6 \to 0^+$, $\ln|t-6| \to -\infty$)

$= -\infty - \ln 8 - \ln 6 + \ln 2 = \boxed{-\infty}$ Diverges

Above, at the PFD step, we did the following Partial Fractions Decomposition:

$$\frac{8}{(x-6)(x+2)} = \frac{A}{x-6}+\frac{B}{x+2}$$

Clearing the denominator yields:

$8 = A(x+2)+B(x-6)$

$8 = Ax+2A+Bx-6B$

$8 = (A+B)x+(2A-6B)$

so that $A+B=0$ and $2A-6B=8$.

The first equation gives $B=-A$, so the second equation gives $2A+6A=8$, which means $8A=8$, so $A=1$. Plugging back into $B=-A$ gives $B=-1$.

Next, complete the following HW problems

Assigned Problems for HW 9

Compute each of the following Integrals. Simplify when possible.

1. $\displaystyle\int_{-\infty}^0 \frac{1}{3-4x}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx$
3. $\displaystyle\int_2^\infty \frac{x}{e^{3x}}\,dx$
4. $\displaystyle\int_e^\infty \frac{\ln x}{x^3}\,dx$
5. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$
6. $\displaystyle\int_e^\infty \frac{1}{x\ln x}\,dx$
7. $\displaystyle\int_{-\infty}^7 \frac{1}{x^2-4x+29}\,dx$
8. $\displaystyle\int_0^5 \frac{6}{x^2-4x-5}\,dx$
9. $\displaystyle\int_0^{e^5} \frac{1}{x\left[25+(\ln x)^2\right]}dx$
10. $\displaystyle\int_1^2 \frac{1}{x\ln x}\,dx$
11. $\displaystyle\int_0^1 x\ln x\,dx$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.