Homework #8

Due ~~**Friday, October 4**~~ in Gradescope by **11:59 pm ET**

Extended to Monday, October 7

- (Optional): **WATCH** Video 4: *Long Division for Partial Fractions*
- **READ** the two worked-out examples in this handout
- **CONSULT** Section 7.4 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 12 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\displaystyle\int \frac{6}{x^2+2x+8}\,dx \overset{\text{complete square}}{=} \int \frac{6}{(x+1)^2+7}\,dx$ $\quad \boxed{\begin{array}{rl} u &= x+1 \\ du &= dx \end{array}}$

$$= 6\int \frac{1}{u^2+7}\,du = 6\left(\frac{1}{\sqrt{7}}\right)\arctan\left(\frac{u}{\sqrt{7}}\right)+C = \boxed{\frac{6}{\sqrt{7}}\arctan\left(\frac{x+1}{\sqrt{7}}\right)+C}$$

Example 2: $\displaystyle\int_{-3}^{1} \frac{6}{x^2+2x-8}\,dx \overset{\text{factor}}{=} \int_{-3}^{1} \frac{6}{(x-2)(x+4)}\,dx \overset{\text{PFD}}{=} \int_{-3}^{1} \frac{1}{x-2}-\frac{1}{x+4}\,dx$

$$= \ln|x-2| - \ln|x+4|\Big|_{-3}^{1} = \big(\ln|-1| - \ln 5\big) - \big(\ln|-5| - \ln 1\big)$$

$$= 0 - \ln 5 - \ln 5 + 0 = \boxed{-2\ln 5}$$

Above, at the PFD step, we did the following Partial Fractions Decomposition:

$$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

Clearing the denominator yields:

$$6 = A(x+4) + B(x-2)$$

$$6 = Ax + 4A + Bx - 2B$$

$$6 = (A+B)x + (4A-2B)$$

so that $A+B=0$ and $4A-2B=6$.

The first equation gives $B=-A$, so the second equation gives $4A+2A=6$, which means $6A=6$, so $A=1$. Plugging back into $B=-A$ gives $B=-1$.

Next, complete the following HW problems

found on the next page

Assigned Problems for HW 8

Exercises 1–12: Compute each of the following Integrals. Simplify when possible

1. $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

Hint on 1: sub $\boxed{x = 3\sec\theta}$

Extra hint on 1: $\dfrac{\tan^2\theta}{\sec^2\theta} = \sin^2\theta$ (Why?)

2. $\displaystyle\int \frac{1}{x^2\sqrt{x^2-16}}\,dx$

Hint on 2: sub $\boxed{x = 4\sec\theta}$

3. $\displaystyle\int \frac{1}{\sqrt{4-4x-x^2}}\,dx$

4. $\displaystyle\int_{-1}^{1} \frac{1}{x^2+4x+7}\,dx$

5. $\displaystyle\int \sqrt{3-2x-x^2}\,dx$

6. $\displaystyle\int \frac{x+4}{x^2+2x+5}\,dx$

7. $\displaystyle\int_3^5 \frac{6}{x^2-4x+7}\,dx$

8. $\displaystyle\int_0^3 \frac{6}{x^2-4x-5}\,dx$

9. $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

10. $\displaystyle\int \frac{\arctan x}{x^2}\,dx = \int x^{-2}\arctan x\,dx$

11. $\displaystyle\int_{\ln 2}^{\ln 5} \frac{2e^x}{e^{2x}-1}\,dx$

Hint on 11:

PFD: $\dfrac{2}{u^2-1} = \dfrac{2}{(u-1)(u+1)}$

$= \dfrac{A}{u-1} + \dfrac{B}{u+1}$

12. $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$

Hint on 12:

PFD: $\dfrac{10}{(x-1)(x^2+9)}$

$= \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+9}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.