Math 121-03, Fall 2024 Professor Rob Benedetto

Homework #7

Due **Friday, September 27** in Gradescope by **11:59 pm ET**

- **READ** the two worked-out examples in this handout
- **CONSULT** Sections 7.2 and 7.3 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 9 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\displaystyle\int \frac{1}{[1+x^2]^{7/2}}\,dx = \int \frac{1}{(1+\tan^2\theta)^{7/2}}\cdot\sec^2\theta\,d\theta = \int \frac{1}{(\sec^2\theta)^{7/2}}\cdot\sec^2\theta\,d\theta$

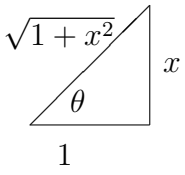
$= \displaystyle\int \frac{1}{(\sqrt{\sec^2\theta})^7}\cdot\sec^2\theta\,d\theta = \int \frac{1}{(\sec\theta)^7}\cdot\sec^2\theta\,d\theta = \int \frac{\sec^2\theta}{\sec^7\theta}\,d\theta = \int \frac{1}{\sec^5\theta}\,d\theta$

$= \displaystyle\int \cos^5\theta\,d\theta = \int \cos^4\theta\cos\theta\,d\theta = \int (1-\sin^2\theta)^2\cos\theta\,d\theta$ $\qquad \boxed{\begin{array}{l} u=\sin\theta \\ du=\cos\theta\,d\theta \end{array}}$

$= \displaystyle\int (1-u^2)^2\,du = \int 1-2u^2+u^4\,du = u-\frac{2u^3}{3}+\frac{u^5}{5}+C = \sin\theta-\frac{2\sin^3\theta}{3}+\frac{\sin^5\theta}{5}+C$

$= \boxed{\displaystyle\frac{x}{\sqrt{1+x^2}}-\frac{2}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^3+\frac{1}{5}\left(\frac{x}{\sqrt{1+x^2}}\right)^5+C}$

Trig Sub above: $\boxed{\begin{array}{l} x=\tan\theta \\ dx=\sec^2\theta d\theta \end{array}}$



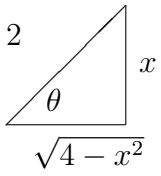
Example 2: $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx = \int \frac{(2\sin\theta)^2}{\sqrt{4-4\sin^2\theta}}\cdot 2\cos\theta\,d\theta = \int \frac{4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}\cdot 2\cos\theta\,d\theta$

$= 4\displaystyle\int \frac{\sin^2\theta}{\sqrt{4}\sqrt{\cos^2\theta}}\cdot 2\cos\theta\,d\theta = 4\int \frac{\sin^2\theta}{2\cos\theta}\cdot 2\cos\theta\,d\theta = 4\int \sin^2\theta\,d\theta$

$= 4\displaystyle\int \frac{1-\cos(2\theta)}{2}\,d\theta = 2\int 1-\cos(2\theta)\,d\theta = 2\left(\theta-\frac{\sin(2\theta)}{2}\right)+C$

$= 2\left(\theta-\frac{2\sin\theta\cos\theta}{2}\right)+C = 2\theta-2\sin\theta\cos\theta+C = \boxed{2\arcsin\left(\frac{x}{2}\right)-\frac{x}{2}\sqrt{4-x^2}+C}$

Trig Sub above: $\boxed{\begin{array}{l} x=2\sin\theta \\ dx=2\cos\theta d\theta \end{array}}$



Next, complete the following HW problems

found on the next page

Assigned Problems for HW 7

Exercises 1–9: Compute each of the following Integrals. Simplify.

1. $\displaystyle\int \sin^2 x \, \cos^3 x \, dx$

2. $\displaystyle\int_0^{\pi/2} \sin^5 x \, dx$

3. $\displaystyle\int_0^{\pi/2} \cos^2 \theta \, d\theta$

4. $\displaystyle\int_0^{\pi/2} \sin^2 x \, \cos^2 x \, dx$

5. $\displaystyle\int x \, \sin^2 x \, dx$

6. $\displaystyle\int_0^1 x^3\sqrt{1-x^2} \, dx$ using Trig Sub

7. $\displaystyle\int \sqrt{9-x^2} \, dx$

8. $\displaystyle\int \frac{1}{(4+x^2)^{5/2}} \, dx$

9. $\displaystyle\int x \, \arcsin x \, dx$

Note: It is also possible to do the integral in problem 6 using the (non-trig) substitution $u = 1 - x^2$. (Can you figure out how to do this?) But in problem 6 above, I am specifically asking you to do it using Trig Sub, so please do it by Trig Sub.

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

~~Friday: 2:00–3:00 pm~~

My Friday 9/27 is moved 30 minutes earlier: 1:30–2:30pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.