Math 121-03, Fall 2024 Professor Rob Benedetto

Homework #6

Due **Wednesday, September 25** in Gradescope by **11:59 pm ET**

- **READ** the two worked-out examples in this handout
- **CONSULT** Sections 6.8 and 7.1 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 10 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\displaystyle\lim_{x\to\infty}\left(e^{1/x}-\frac{4}{x}\right)^x \overset{1^\infty}{=} \lim_{x\to\infty}\exp\left(\ln\left(\left(e^{1/x}-\frac{4}{x}\right)^x\right)\right)$

$$= \exp\left(\lim_{x\to\infty}\ln\left[\left(e^{1/x}-\frac{4}{x}\right)^x\right]\right) = \exp\left(\lim_{x\to\infty} x\ln\left(e^{1/x}-\frac{4}{x}\right)\right)$$

$$\overset{\infty\cdot 0}{=} \exp\left(\lim_{x\to\infty}\frac{\ln\left(e^{1/x}-\frac{4}{x}\right)}{\frac{1}{x}}\right) \overset{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} \exp\left(\lim_{x\to\infty}\frac{\left(\frac{1}{e^{1/x}-\frac{4}{x}}\right)\cdot\left[e^{1/x}\left(-\frac{1}{x^2}\right)+\frac{4}{x^2}\right]}{-\frac{1}{x^2}}\right)$$

$$= \exp\left[\lim_{x\to\infty}\left(\frac{1}{e^{1/x}-\frac{4}{x}}\right)\cdot\left[e^{1/x}\left(-\frac{1}{\cancel{x^2}}\right)+\frac{4}{\cancel{x^2}}\right](-\cancel{x^2})\right]$$

$$= \exp\left[\lim_{x\to\infty}\left(\frac{1}{\cancel{e^{1/x}}^{\,1}-\cancel{\frac{4}{x}}^{\,0}}\right)\cdot\left(\cancel{e^{1/x}}^{\,1}(1)-4\right)\right] = \exp\big(1\cdot(-3)\big) = \boxed{e^{-3}}$$

Example 2: $\displaystyle\int \arctan\left(\frac{1}{x}\right)\,dx$

$u = \arctan\left(\frac{1}{x}\right)$	$dv = 1dx$
$du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\left(-\frac{1}{x^2}\right)dx$	$v = x$
$du = -\frac{1}{x^2+1}dx$	$\leftarrow$ simplify

$$= x\arctan\left(\frac{1}{x}\right) - \int\frac{-x}{x^2+1}\,dx$$

w	$= x^2+1$
dw	$= 2x\,dx$
$\frac{1}{2}dw$	$= x\,dx$

$$= x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\int\frac{1}{w}\,dw = x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\ln|w| + C =$$

$$= x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2+1| + C = \boxed{x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\ln(x^2+1) + C}$$

Next, complete the following HW problems

found on the next page

Assigned Problems for HW 6

Exercises 1–3: Compute each of the following Limits. Simplify. *Justify every step.*

1. $\lim_{x\to\infty} \dfrac{\ln\left(5+e^{3x}\right)}{x}$

2. $\lim_{x\to\infty} \left(\dfrac{x}{x+1}\right)^{x}$

3. $\lim_{x\to\infty} \left(e^{1/x^6} - \dfrac{6}{x^6}\right)^{x^6}$

Exercises 4–10: Compute each of the following Integrals using Integration by Parts. Simplify. *Justify every step.*

4. $\displaystyle\int x\cos(5x)\ dx$

5. $\displaystyle\int_0^1 \arctan x\ dx$

6. $\displaystyle\int_0^5 \frac{x^2}{e^x}\ dx$

7. $\displaystyle\int (\ln x)^2\ dx$

8. $\displaystyle\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)\ dx$

9. $\displaystyle\int x\arctan x\ dx$

10. $\displaystyle\int \ln\left(x^2+7\right)\ dx$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.