Homework #5

Due **Friday, September 20** in Gradescope by **11:59 pm ET**

- **READ** the two worked-out examples in this handout
- **CONSULT** Section 6.8 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 18 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\lim\limits_{x\to 0} \dfrac{\arcsin x + \cos(3x) - e^x}{\arctan(3x) + x^2 - \sin(3x)} \overset{\left(\frac{0}{0}\right)}{=} \overset{\text{L'H}}{\lim\limits_{x\to 0}} \dfrac{\frac{1}{\sqrt{1-x^2}} - 3\sin(3x) - e^x}{\frac{1}{1+(3x)^2}\cdot(3) + 2x - 3\cos(3x)}$

$\overset{\text{prep}}{=} \lim\limits_{x\to 0} \dfrac{(1-x^2)^{-1/2} - 3\sin(3x) - e^x}{3(1+9x^2)^{-1} + 2x - 3\cos(3x)} \overset{\left(\frac{0}{0}\right)}{\overset{\text{L'H}}{=}} \lim\limits_{x\to 0} \dfrac{-\frac{1}{2}(1-x^2)^{-3/2}\cdot(-2x) - 9\cos(3x) - e^x}{-3(1+9x^2)^{-2}\cdot(18x) + 2 + 9\sin(3x)}$

$\overset{\text{rewrite}}{=} \lim\limits_{x\to 0} \dfrac{\frac{x}{(1-x^2)^{3/2}} - 9\cos(3x) - e^x}{\frac{-54x^2}{(1+9x^2)} + 2 + 9\sin(3x)} = \dfrac{-9-1}{2} = \dfrac{-10}{2} = \boxed{-5}$

Example 2: $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x^3}\right)^{x^3} \overset{1^\infty}{=} \lim\limits_{x\to\infty} e^{\ln\left(\left(1-\frac{2}{x^3}\right)^{x^3}\right)} \overset{\infty\cdot 0}{=} e^{\lim\limits_{x\to\infty} x^3\ln\left(1-\frac{2}{x^3}\right)}$

$= e^{\lim\limits_{x\to\infty} \frac{\ln\left(1-\frac{2}{x^3}\right)}{\frac{1}{x^3}}} \overset{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} e^{\lim\limits_{x\to\infty} \frac{\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)}{-\frac{3}{x^4}}} = e^{\lim\limits_{x\to\infty}\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)\cdot\left(-\frac{x^4}{3}\right)}$

$= e^{\lim\limits_{x\to\infty}\left(\frac{1}{1-\frac{2}{x^3}}\right)(-2)} = e^{1\cdot(-2)} = \boxed{e^{-2}}$

Next, complete the following HW problems

found on the next page

Assigned Problems for HW 5

Exercises 1–18: Compute each of the following Limits. Simplify. *Justify every step.*

1. $\lim_{\theta \to \pi/2} \dfrac{1 - \sin\theta}{1 + \cos(2\theta)}$

2. $\lim_{x \to \infty} \dfrac{\ln x}{\sqrt{x}}$

3. $\lim_{x \to 0^+} \dfrac{\ln x}{x}$

4. $\lim_{x \to 0} \dfrac{e^{2x} - 1 - 2x}{x^2}$

5. $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

6. $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

7. $\lim_{x \to 0} \dfrac{\arcsin(3x)}{\arctan(4x)}$

8. $\lim_{x \to 0} \dfrac{x - \arcsin x}{\arctan(2x) - 2x}$

9. $\lim_{x \to 0} \dfrac{3xe^x - \arctan(3x)}{x + \ln(1 - x)}$

10. $\lim_{x \to 0} \dfrac{\arcsin x + x^2 - x}{\cos x - \arctan(5x) - e^{-5x}}$

11. $\lim_{x \to \infty} x \sin\left(\dfrac{\pi}{x}\right)$

12. $\lim_{x \to \infty} x \ln\left(1 - \dfrac{1}{x}\right)$

13. $\lim_{x \to 0^+} x \ln x$

14. $\lim_{x \to 0^+} \sqrt{x} \ln x$

15. $\lim_{x \to \infty} x^2 e^{-x}$

16. $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$

17. $\lim_{x \to 0^+} (1 + \ln(1 - 3x))^{1/x}$

18. $\lim_{x \to \infty} \left(1 - \arctan\left(\dfrac{7}{x^4}\right)\right)^{x^4}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

~~Thursday: 1:30–3:00 pm~~

My Office Hours Thursday 9/19 are EXTENDED: 1:30–4:00pm

~~Friday: 2:00–3:00 pm~~

My Office Hour Friday 9/20 is CANCELLED

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	~~7:30–9:00pm:~~	**Natalie** Stott
	Wed 9/11 and Wed 9/18 only: 8:15–9:45pm	
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.