Homework #4

Due **Wednesday, September 18** in Gradescope by **11:59 pm ET**

- **WATCH** Video 3: *Limits Review*
- **READ** the three worked-out examples in this handout
- **WRITE AND SUBMIT** solutions to the 21 assigned problems in this handout

NOTE: Show your work, as always.

Example 1: $\displaystyle\int_e^{e^3} \frac{1}{x\left(3+(\ln x)^2\right)}\,dx$

$$\begin{array}{rl} u &= \ln x \\ du &= \dfrac{1}{x}\,dx \end{array} \qquad \begin{array}{rl} x=e & \to u=\ln e=1 \\ x=e^3 & \to u=\ln e^3=3 \end{array}$$

$$= \int_1^3 \frac{1}{3+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{\sqrt{3}}\arctan\left(\frac{u}{\sqrt{3}}\right)\Big|_1^3 = \frac{1}{\sqrt{3}}\left(\arctan\left(\frac{3}{\sqrt{3}}\right)-\arctan\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}\left(\frac{2\pi}{6}-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{6\sqrt{3}}}$$

Example 2: $\displaystyle\int \frac{e^{3x}}{4+e^{3x}}\,dx$

$$\begin{array}{rl} u &= 4+e^{3x} \\ du &= 3e^{3x}\,dx \\ \dfrac{1}{3}du &= e^{3x}\,dx \end{array} \qquad = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u|+C$$

$$= \boxed{\frac{1}{3}\ln\left|4+e^{3x}\right|+C}$$

Example 3: $\displaystyle\int \frac{e^{3x}}{4+e^{6x}}\,dx = \int \frac{e^{3x}}{4+(e^{3x})^2}\,dx$

[Note: let's look for a perfect square to get ready for arctan !!]

$$\begin{array}{rl} u &= e^{3x} \\ du &= 3e^{3x}\,dx \\ \dfrac{1}{3}du &= e^{3x}\,dx \end{array}$$

$$= \frac{1}{3}\int \frac{1}{4+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{3}\left(\frac{1}{2}\arctan\left(\frac{u}{2}\right)\right)+C$$

$$= \boxed{\frac{1}{6}\arctan\left(\frac{e^{3x}}{2}\right)+C}$$

Next, complete the following HW problems

found on the next page

Assigned Problems for HW 4

Exercises 1–10: Compute each of the following Integrals. Simplify.

1. $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}}\,dx$
2. $\displaystyle\int_0^{\ln 3} \frac{e^x}{3+e^{2x}}\,dx$
3. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$
4. $\displaystyle\int_4^{4\sqrt{3}} \frac{1}{16+x^2}\,dx$
5. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$
6. $\displaystyle\int \frac{x^2}{x^2+4}\,dx$
7. $\displaystyle\int \frac{2x^2+5}{x^2+1}\,dx$
8. $\displaystyle\int \frac{1}{(1+x^2)\,(5+(\arctan x)^2)}\,dx$
9. $\displaystyle\int_3^9 \frac{1}{\sqrt{x}\,(x+9)}\,dx$
10. $\displaystyle\int \frac{x^2+x+1}{x^2+4}\,dx$

Exercises 12–20: Compute each of the following Limits. Simplify. Use arrows to justify the size arguments.

11. $\displaystyle\lim_{x\to 5^+} \frac{1}{x-5}$
12. $\displaystyle\lim_{x\to 5^-} \frac{1}{x-5}$
13. $\displaystyle\lim_{x\to 8^+} \ln|x-8|$
14. $\displaystyle\lim_{x\to 8^-} \ln|x-8|$
15. $\displaystyle\lim_{x\to 3^+} e^{2/(x-3)}$
16. $\displaystyle\lim_{x\to 3^-} e^{2/(x-3)}$
17. $\displaystyle\lim_{x\to\infty} \ln\left(1-\arctan\left(\frac{5}{x^4}\right)\right)$
18. $\displaystyle\lim_{x\to\infty} \ln\left(\frac{\pi}{2}-\arctan x\right)$
19. $\displaystyle\lim_{x\to 4^-} \ln\big|\ln|x-4|\big|$
20. $\displaystyle\lim_{x\to 0^+} \arctan\left(\frac{\ln x}{5}\right)$

21. Present two different methods to Prove that $\displaystyle\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

(Use a LIDS proof for one, and integration methods for the other. Do *not* use an a-rule.)

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm
Thursday: 1:30-3:00 pm
Friday: 2:00–3:00 pm
(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	~~7:30–9:00pm:~~	**Natalie** Stott
	Wed 9/11 and Wed 9/18 only: 8:15–9:45pm	
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.