Math 121-03, Fall 2024 — Professor Rob Benedetto

Homework #16

Due **Friday, November 15** in Gradescope by **11:59 pm ET**

- **REVIEW** your class notes about power series representations of functions
- **CONSULT** Section 11.9 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 21 assigned problems in this handout

NOTE: Show your work, as always.

Assigned Problems for HW 16

Exercises 1–8: Find the Series Representation for the following functions using Substitution into a known series, and determine the Radius of Convergence R. Simplify.

1. $\dfrac{1}{1+x^2}$
2. $\dfrac{x^2}{x^4+16}$
3. $x^3\cos\left(x^2\right)$
4. $5x^2\sin\left(5x\right)$
5. $\dfrac{d}{dx}\left(x^3\arctan(7x)\right)$
6. $\displaystyle\int x^3\arctan(7x)\ dx$
7. $\dfrac{d}{dx}x^2\ln(1+6x)$
8. $\displaystyle\int x^4e^{-x^3}\ dx$

Exercise 9: Find the Series Representation for $f(x) = \dfrac{1}{(1+x)^2}$

write as pwr ser?

Hint: $$\frac{1}{(1+x)^2} = \frac{d}{dx}\left(-\frac{1}{1+x}\right) = \dots$$

Exercise 10: Prove the Power Series Representation formula for $\arctan x$, as shown in class. Yes, I did it out in class (and you should consult your notes to see how), and yes, this includes showing that $C=0$.

Exercise 11: Find Series Representation for $\ln(5-x)$. Solve for C and the Radius R.

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Hint: $$\ln(5-x) = \int \frac{-1}{5-x}\ dx = \int \frac{-1}{5\left(1-\frac{x}{5}\right)}\ dx = -\frac{1}{5}\int \frac{1}{1-\frac{x}{5}}\ dx = \dots$$

Exercise 12: Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods.
First, using the *Definition* of the MacLaurin Series ("Chart Method").
Second, use Substitution into a known series. Your answers should be in Sigma notation.

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Exercise 13: In this problem, you do **not** need to state the Radius. Your answers here should be in Sigma notation $\sum_{n=0}^{\infty}$. You may use the fact that $\boxed{\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification. Here goes:

(a) Use the Definition ("Chart Method") to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.
Hints: yes, you should solve for $+C$. yes, C should equal 1. Show why $C = 1$.

Exercises 14–21: Find the Sum of each of the following Series.
Please note: I'm not asking you to run convergence tests on them, because I'm already telling you that they converge! Instead, I'm asking you to **Find the Sum** of each of the following convergent series. (And of course, justify your answers / show you work, as always.)

14. $\displaystyle\sum_{n=0}^{\infty} \frac{7^n}{n!}$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ 5^n}{n!}$

16. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

17. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{6^{2n}\ (2n)!}$

18. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n\ n!}$

19. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

20. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$

21. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.