Homework #15

Due **Friday, November 8** in Gradescope by **11:59 pm ET**

- **READ** the three worked-out examples in this handout
- **CONSULT** Section 11.8 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 11 assigned problems in this handout

NOTE: Show your work, as always.

In each of the following examples, determine the Interval and Radius of Convergence. Justify.

Example 1: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(5x-2)^n}{(n+5)\,8^n}$ Use Ratio Test. $L = \displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

$$= \lim_{n\to\infty}\left|\frac{\dfrac{\cancel{(-1)^{n+1}}(5x-2)^{n+1}}{(n+6)8^{n+1}}}{\dfrac{\cancel{(-1)^n}(5x-2)^n}{(n+5)8^n}}\right| = \lim_{n\to\infty}\left|\frac{(5x-2)^{n+1}}{(5x-2)^n}\right| \cdot \left(\cancelto{1}{\frac{n+5}{n+6}}\right) \cdot \frac{8^n}{8^{n+1}} = \frac{|5x-2|}{8}$$

The Ratio Test gives convergence for x when $\dfrac{|5x-2|}{8} < 1$ or $|5x-2| < 8$.

That is, $-8 < 5x-2 < 8 \Longrightarrow -6 < 5x < 10 \Longrightarrow -\frac{6}{5} < x < 2$

Manually Test Endpoints: (where $L=1$ and Ratio Test is Inconclusive)

- $x = 2$ The original series becomes $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(5(2)-2)^n}{(n+5)\,8^n} = \sum_{n=1}^{\infty} \frac{(-1)^n\cancel{8^n}}{(n+5)\,\cancel{8^n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$

which is Convergent by AST, because:

1. $b_n = \dfrac{1}{n+5} > 0$, 2. $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{n+5} = 0$, and

3. terms decreasing: $b_{n+1} = \dfrac{1}{n+6} < \dfrac{1}{n+5} = b_n$

So $x = 2$ is in the Domain.

- $x = -\dfrac{6}{5}$ The original series becomes $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\left(5\left(-\frac{6}{5}\right)-2\right)^n}{(n+5)\,8^n} = \sum_{n=1}^{\infty}\frac{(-1)^n(-8)^n}{(n+5)\,8^n}$

$$= \sum_{n=1}^{\infty}\frac{(-1)^n(-1)^n\cancel{8^n}}{(n+5)\cancel{8^n}} = \sum_{n=1}^{\infty}\frac{\cancelto{1}{(-1)^{2n}}}{n+5} = \sum_{n=1}^{\infty}\frac{1}{n+5} \approx \sum_{n=1}^{\infty}\frac{1}{n} \text{ the Div Harmonic } p\text{-Series } p=1.$$

LCT: $\displaystyle\lim_{n\to\infty}\frac{\frac{1}{n+5}}{\frac{1}{n}} = \lim_{n\to\infty}\cancelto{1}{\frac{n}{n+5}} = 1$ which is *Finite* and *Non-zero*. Therefore, $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+5}$ is also Divergent by LCT

So $x = -\dfrac{6}{5}$ is NOT in the Domain.

To sum up: the Interval of Convergence is $\boxed{I = \left(-\dfrac{6}{5}, 2\right]}$

and the Radius of Convergence is $\boxed{R = \dfrac{8}{5}}$

Example 2: $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ Use Ratio Test.

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\dfrac{x^{2(n+1)+1}}{(2(n+1)+1)!}}{\dfrac{x^{2n+1}}{(2n+1)!}} \right| = \lim_{n\to\infty} \left|\frac{x^{2n+3}}{x^{2n+1}}\right| \frac{(2n+1)!}{(2n+3)!}$$

$$= \lim_{n\to\infty} \frac{x^2}{\underbrace{(2n+3)(2n+2)}_{\to\infty}} = 0 < 1$$

Converges by the Ratio Test for all x. So $\boxed{I = (-\infty, \infty)}$ with $\boxed{R = \infty}$.

Example 3: $\displaystyle\sum_{n=0}^{\infty} n^n (x-7)^n$ Use Ratio Test.

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^{n+1}(x-7)^{n+1}}{n^n (x-7)^n}\right| = \lim_{n\to\infty} \underbrace{\frac{(n+1)^n}{n^n}}_{\to e} \underbrace{(n+1)}_{\to\infty} |x-7| = \infty > 1$$

Diverges by the Ratio Test for all x *unless* $x - 7 = 0$ or $x = 7$. So $\boxed{I = \{7\}}$ with $\boxed{R = 0}$.

Next, complete the following problems:

Assigned Problems for HW 15

Exercises 1–8: Determine the Interval and Radius of Convergence for each of the following Power Series. Use the Ratio Test and manually check convergence at the Endpoints for the Finite Intervals. Follow the examples above for statements/format for all three cases.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 \cdot 4^n}$
3. $\displaystyle\sum_{n=1}^{\infty} n!\ \ln n\ (x-6)^n$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (9x-4)^n}{n^8 \cdot 5^n}$
5. $\displaystyle\sum_{n=0}^{\infty} (3n)!\ (2x-1)^n$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (6x+1)^n}{(6n+1)\cdot 7^n}$
7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ x^{2n}}{(2n)!}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (3x-5)^n}{(n+6)^2 \cdot 7^{n+1}}$

Exercises 9–11:
Find the Power Series Representation for the following functions and determine the Interval of Convergence.

9. $f(x) = \dfrac{1}{1+x}$
10. $f(x) = \dfrac{5}{1-4x}$
11. $f(x) = \dfrac{1}{3-x}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.