Math 121-03, Fall 2024 — Professor Rob Benedetto

Homework #14

Due **Wednesday, November 6** in Gradescope by **11:59 pm ET**

- **READ** the two worked-out examples in this handout
- **CONSULT** Section 11.6 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 11 assigned problems in this handout

NOTE: Show your work, as always.

In each of the following examples, determine whether the Series is Absolutely Convergent, Conditionally Convergent, or Divergent. Justify with any Convergence Test(s).

Example 1: $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+7}{n^7+2}$ Abs.Ser. is $\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^5}$

Now $\sum \frac{1}{n^5}$ converges by p-Test: $p = 5 > 1$.

Prep LCT: $\lim_{n\to\infty} \frac{\frac{n^2+7}{n^7+2}}{\frac{1}{n^5}} = \lim_{n\to\infty} \frac{n^7+7n^5}{n^7+2} \cdot \frac{\frac{1}{n^7}}{\frac{1}{n^7}} = \lim_{n\to\infty} \frac{1+\frac{7}{n^2}}{1+\frac{2}{n^7}} = 1$ Finite & Nonzero.

So the **Absolute Series** $\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2}$ also **Converges** by LCT

So, finally, the Original Series is Absolutely Convergent (A.C.) (by Definition).

Example 2: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n+3}$ Abs.Ser. is $\sum_{n=1}^{\infty} \frac{1}{7n+3} \approx \sum_{n=1}^{\infty} \frac{1}{n}$

Now $\sum \frac{1}{n}$ diverges by p-Test: $p = 1$.

Prep LCT: $\lim_{n\to\infty} \frac{\frac{1}{7n+3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{7n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{7+\frac{3}{n}} = \frac{1}{7}$ Finite & Nonzero.

So the **Absolute Series** $\sum_{n=1}^{\infty} \frac{1}{7n+3}$ also **Diverges** by Limit Comparison Test.

Now examine the original alternating series with the Alternating Series Test.

- Isolate $b_n = \frac{1}{7n+3} > 0$ and
- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{7n+3} = 0$ (with $7n+3 \to \infty$) and
- Terms Decreasing: $b_{n+1} < b_n$ because $b_{n+1} = \frac{1}{7(n+1)+3} = \frac{1}{7n+10} < \frac{1}{7n+3} = b_n$

Therefore, the **Original Series Converges** by the Alternating Series Test.

So, finally, the Original Series is Conditionally Convergent (C.C.) (by Definition).

Next, complete the HW problems found on next page

Assigned Problems for HW 14

Exercises 1–5: Determine whether the given series is Absolutely Convergent (AC), Conditionally Convergent (CC), or Divergent.
(Number 3 can be done with the Ratio Test, but use the AC and CC charts for 1, 2, 4, 5.)

1. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3+8}{n^8+3}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (\ln n) \, (2n)!}{n^n \, 2^{3n} \, (n!)}$
4. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2}$
5. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3}{n^7+2}$

Exercise 6: Write the statement of the Absolute Convergence Test.

Exercise 7: Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ Converges.

Exercise 8: Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin^2 n}{n^8+2}$ Converges.

Exercises 9–11: Various Review; as always, justify all steps.

9. Show that the Sequence $\left\{ \left(\dfrac{n}{n+1} \right)^n \right\}_{n=1}^{\infty}$ converges to $\dfrac{1}{e}$.

10. Determine whether the Series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$ converges or diverges.

11. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, 5^{2n+1}}{2^{5n-1}}$.

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.