Math 121-03, Fall 2024 Professor Rob Benedetto

Homework #13

Due **Friday, October 25** in Gradescope by **11:59 pm ET**

- $\boxed{\textbf{READ} \text{ the two worked-out examples in this handout}}$
- $\boxed{\textbf{CONSULT} \text{ Sections 11.5 and 11.6 of the Stewart Calculus textbook}}$
- $\boxed{\textbf{WRITE AND SUBMIT} \text{ solutions to the 11 assigned problems in this handout}}$

NOTE: Show your work, as always.

In each of the following examples, determine whether the given series Converges Absolutely or Diverges. Justify, as always.

Example 1: $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n! \cdot 2^n}$ Try Ratio Test:

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^{n+1}}{(n+1)!2^{n+1}}}{\frac{n^n}{n!\cdot 2^n}} \right| = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{n^n} \cdot \frac{n!}{(n+1)!} \cdot \frac{2^n}{2^{n+1}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^n \cancel{(n+1)}}{n^n} \cdot \frac{\cancel{n!}}{\cancel{(n+1)}\cancel{n!}} \cdot \frac{\cancel{2^n}}{\cancel{2^n}\cdot 2} = \lim_{n\to\infty} \cancel{\left(\frac{n+1}{n}\right)^n} \overset{e}{\nearrow} \cdot \left(\frac{1}{2}\right) = \frac{e}{2} > 1$$

So the Original Series $\boxed{\textbf{Diverges by the Ratio Test}}$

Example 2: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{e^{2n} \cdot\ n! \cdot\ n^n}$ Try Ratio Test: $L = \displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$= \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(2(n+1))!}{e^{2(n+1)}\ (n+1)!\ (n+1)^{n+1}}}{\frac{(-1)^n(2n)!}{e^{2n}\ n!\ n^n}} \right| = \lim_{n\to\infty} \frac{(2n+2)!}{(2n)!} \cdot \frac{n^n}{(n+1)^{n+1}} \cdot \frac{e^{2n}}{e^{2n+2}} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)(2n)!}{(2n)!} \cdot \frac{n^n}{(n+1)^n(n+1)} \cdot \frac{\cancel{e^{2n}}}{\cancel{e^{2n}}e^2} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{(2\cancel{(n+1)})(2n+1)}{\cancel{(n+1)}(n+1)} \cdot \cancel{\left(\frac{n}{n+1}\right)^n} \overset{\frac{1}{e}}{\nearrow} \cdot \frac{1}{e^2} = \lim_{n\to\infty} \frac{2n+1}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \cdot \left(\frac{2}{e^3}\right)$$

$$= \lim_{n\to\infty} \frac{2 + \cancel{\frac{1}{n}}^{\,0}}{1 + \cancel{\frac{1}{n}}^{\,0}} \cdot \left(\frac{2}{e^3}\right) = 2\left(\frac{2}{e^3}\right) = \frac{4}{e^3} < 1$$

So the Original Series $\boxed{\textbf{Converges Absolutely (AC) by the Ratio Test}}$

Next, complete the HW problems found on next page

Assigned Problems for HW 13

Exercise 1: Consider $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+4n+7}$.
Use **two** Different methods, namely the Integral Test and the Limit Comparison Test, to prove that this series Diverges.

Note: *You may skip checking the Integral Test preconditions here this time. yay!*

Exercise 2: Determine whether the Alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n+1}$ Converges or Diverges.

Exercises 3–8: Determine whether the given series is Absolutely Convergent or Divergent.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{5^n}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{100^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{100}\ 100^n}{n!}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

Exercise 9: Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

9(a) Show that n^{th} Term Divergence Test is **Inconclusive** for this series.

9(b) Show that the Ratio Test is **Inconclusive** for this series.

9(c) Show that the series Diverges using the Integral Test.
Skip checking the 3 preconditions here.
(**Note:** This is an example where the terms approach 0 but the series Diverges.)

Exercise 10: Prove that $\displaystyle\sum_{n=1}^{\infty} \frac{6}{n^6}$ is Convergent by using the Limit Comparison Test.

(**Note**: this work will be a sample proof of the fact that *Constant multiple of a Convergent series* **is** *Convergent.*)

Exercise 11: Show that $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ Diverges using **two** different methods.

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

~~Friday: 2:00–3:00 pm~~

My Friday 10/25 is moved 30 minutes earlier: 1:30–2:30pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.