Math 121-03, Fall 2024 | Professor Rob Benedetto

Homework #12

Due **Wednesday, October 23** in Gradescope by **11:59 pm ET**

- **READ** the three worked-out examples in this handout
- **CONSULT** Sections 11.3 and 11.4 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 16 assigned problems in this handout

NOTE: Show your work, as always.

In each of the following examples, determine whether the given series Converges or Diverges. Justify with any Convergence Test(s).

Example 1: $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ Integral Test: Related Function $f(x) = \frac{\ln x}{x}$ is:

continuous $(x > 0)$; positive $(x > 1)$; decreasing $(x > e)$ since $f'(x) = \frac{1 - \ln x}{x^2} < 0$ for $x > e$.

Study the Related Integral

$$\int_2^{\infty} \frac{\ln x}{x}\, dx = \lim_{t\to\infty} \int_2^t \frac{\ln x}{x}\, dx \qquad \boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\, dx \end{array}} \qquad \boxed{\begin{array}{l} x = 2 \Rightarrow u = \ln 2 \\ x = t \Rightarrow u = \ln t \end{array}}$$

$$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} u\, du = \lim_{t\to\infty} \left.\frac{u^2}{2}\right|_{\ln 2}^{\ln t} = \lim_{t\to\infty} \frac{\overbrace{(\ln t)^2}^{\infty}}{2} - \frac{(\ln 2)^2}{2} = \boxed{\infty} \qquad \text{Integral diverges}$$

So the Original Series also **Diverges by the Integral Test**

Example 2: $\sum_{n=1}^{\infty} \frac{1}{n^3 + 7} \approx \sum_{n=1}^{\infty} \frac{1}{n^3}$ ← Comparison Series converges by p-Test: $p = 3 > 1$

Bound/Compare Terms: $0 \le \frac{1}{n^3 + 7} \le \frac{1}{n^3}$

Therefore, the Original Series also **Converges by the Comparison Test**

Example 3: $\sum_{n=1}^{\infty} \frac{n^3 + 2}{n^4 + 8} \approx \sum_{n=1}^{\infty} \frac{n^3}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n}$ ← Comparison Series diverges by p-Test: $p = 1$

Next check: $\lim_{n\to\infty} \frac{\frac{n^3+2}{n^4+8}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^4 + 2n}{n^4 + 8} \cdot \frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)} = \lim_{n\to\infty} \frac{1 + \overbrace{\frac{2}{n^3}}^{0}}{1 + \underbrace{\frac{8}{n^4}}_{0}} = 1$ Finite and Nonzero

Therefore, the Original Series also **Diverges by the Limit Comparison Test**

Next, complete the HW problems found on next page

Assigned Problems for HW 12

Exercises 1–4: Use the Integral Test to determine whether the given series Converges or Diverges. You do **NOT** need to check the 3 pre-conditions for the Integral Test this time.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ 2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ 3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ 4. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

Exercise 5: Consider $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$. Use **two** different methods, namely the Integral Test (no pre-condition check needed) and the Comparison Test, to prove that this series Converges.

Exercises 6–9: Determine whether the series Converges or Diverges using either the Comparison Test **OR** the Limit Comparison Test.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ 7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+5}{n^3}$ 8. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$ 9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2}$

Exercise 10: Consider $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+n}{n^4}$. Use **two** different methods to prove that this series Converges. Use the Limit Comparison Test and then a *split-split* algebra technique into p-series pieces.

Exercises 11–16: Determine whether the given series Converges or Diverges. Justify.
For some of these, it may be useful to use earlier tests we have learned, not just the integral or comparison tests.

11. $\displaystyle\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi n^4+1}{6n^4+5}\right)$ 12. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(\pi n^4+1)}{6n^4+5}$ 13. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{n^9}+\frac{7^n}{9^n}$

14. $\displaystyle\sum_{n=1}^{\infty} n^6+6$ 15. $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+6}{n^6+1}$ 16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6+1}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.