Math 121-03, Fall 2024 Professor Rob Benedetto

Homework #11

Due **Friday, October 18** in Gradescope by **11:59 pm ET**

- **READ** the five worked-out examples in this handout
- **CONSULT** Section 11.2 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 19 assigned problems in this handout

NOTE: Show your work, as always.

In each of the following examples, determine whether the given series Converges or Diverges. If it converges, find the value of the Sum. Justify, as always.

Example 1: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ 5^{n-1}}{3^{2n+1}} = -\frac{1}{3^3}+\frac{5}{3^5}-\frac{5^2}{3^7}+\frac{5^3}{3^9}+\cdots$ Here $a = -\frac{1}{27}$ and $r = -\frac{5}{3^2} = -\frac{5}{9}$.

Series **Converges by Geometric Series Test (GST)**, because $|r| = \left|-\frac{5}{9}\right| = \frac{5}{9} < 1$ with

$$\text{SUM}= \frac{a}{1-r} = \frac{-\frac{1}{27}}{1-\left(-\frac{5}{9}\right)} = \frac{-\frac{1}{27}}{\frac{14}{9}} = -\frac{1}{\cancel{27}_3}\cdot\frac{\cancel{9}^{\,1}}{14} = -\frac{1}{3}\cdot\frac{1}{14} = \boxed{-\frac{1}{42}}$$

Example 2: $\displaystyle\sum_{n=0}^{\infty}\left(\frac{7}{3}\right)^n = 1+\frac{7}{3}+\frac{7^2}{3^2}+\frac{7^3}{3^3}+\cdots$ Here $a = 1$ and $r = \frac{7}{3}$.

Series **Diverges by GST**, because $|r| = \frac{7}{3} \geq 1$.

Example 3: $\sum_{n=1}^{\infty}\frac{e^n}{n^2}$ **Diverges by the n^{th} Term Divergence Test (nTDT)** because

$$\lim_{n\to\infty}\frac{e^n}{n^2} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty}\frac{e^x}{x^2} \overset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim_{x\to\infty}\frac{e^x}{2x} \overset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim_{x\to\infty}\frac{\cancel{e^x}^{\,\infty}}{2} = \infty \neq 0$$

Example 4: $\displaystyle\sum_{n=1}^{\infty} 3$ **Diverges by nTDT** because $\lim_{n\to\infty} 3 = 3 \neq 0$

[Q: Is this also Geometric? If so, r =?]

Example 5: $\displaystyle\sum_{n=1}^{\infty} e^{1/n}$ **Diverges by nTDT** because $\lim_{n\to\infty} \cancel{e^{1/n}}^{\,1} = 1 \neq 0$

Next, complete the HW problems found on next page

Assigned Problems for HW 11

Exercises 1–4: Determine whether each of the following Converge or Diverge. Pay attention to sequences vs. series. Justify as always.

1. $\{8\}_{n=1}^{\infty}$
2. $\displaystyle\sum_{n=1}^{\infty} 8$
3. $\left\{\dfrac{2n}{3n+1}\right\}_{n=1}^{\infty}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{3n+1}$

Exercises 5–17: Determine whether the given series Converges or Diverges. If it converges, find the value of the Sum. Justify as always.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{8}{5^n}$
6. $\displaystyle\sum_{n=0}^{\infty} \frac{8}{5^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{9^{n-1}}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{7^{n+1}}{3^n}$
9. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4^{2n+1}}{3^{3n-1}}$
10. $\displaystyle\sum_{n=1}^{\infty} e^n$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$
12. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(1999)^n}$
13. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1999}$
14. $\displaystyle\sum_{n=1}^{\infty} \arctan n$
15. $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$
16. $\displaystyle\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi n^4+1}{3n^4+5}\right)$
17. $\displaystyle\sum_{n=1}^{\infty} \left(1+\ln\left(1+\frac{5}{n}\right)\right)^n$

Exercises 18–19 Consider these variable versions of Geometric Series. Find the values of x for which the series Converges. Find the sum of the Series for those values of x (answer in terms of x).

18. $\displaystyle\sum_{n=1}^{\infty} (-5)^n x^n$
19. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.