Homework #10

Due **Friday, October 11** in Gradescope by **11:59 pm ET**

- **READ** the five worked-out examples in this handout
- **CONSULT** Section 11.1 of the Stewart Calculus textbook
- **WRITE AND SUBMIT** solutions to the 20 assigned problems in this handout

NOTE: Show your work, as always.

In each of the following examples, determine whether the given sequence Converges or Diverges. If it converges, find the Limit.

Example 1: $\left\{\dfrac{\ln n}{n^3}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\frac{\ln n}{n^3}\overset{\frac{\infty}{\infty}}{=}\lim_{x\to\infty}\frac{\ln x}{x^3}\overset{\text{L'H}}{=}\lim_{x\to\infty}\frac{x^{-1}}{3x^2}=\lim_{x\to\infty}\frac{1}{3x^3}=\boxed{0}$$ Converges

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Example 2: $\left\{\dfrac{e^n}{n^2}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\frac{e^n}{n^2}\overset{\frac{\infty}{\infty}}{=}\lim_{x\to\infty}\frac{e^x}{x^2}\overset{\text{L'H}}{=}\lim_{x\to\infty}\frac{e^x}{2x}=\lim_{x\to\infty}\frac{e^x}{2}=\boxed{\infty}$$ Diverges

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Example 3: $\left\{\dfrac{4-9n^3}{5n^3+8n^2-7n-6}\right\}_{n=1}^{\infty}$

Note: We could switch n to x and use L'H (3 times), BUT this is easier:

$$\lim_{n\to\infty}\frac{4-9n^3}{5n^3+8n^2-7n-6}\cdot\frac{\left(\frac{1}{n^3}\right)}{\left(\frac{1}{n^3}\right)}=\lim_{n\to\infty}\frac{\frac{4}{n^3}-9}{5+\frac{8}{n}-\frac{7}{n^2}-\frac{6}{n^3}}\overset{0}{=}\boxed{-\frac{9}{5}}$$ Converges

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Example 4: $\left\{\left(1-\sin\left(\dfrac{6}{n^3}\right)\right)^{n^3}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\left(1-\sin\left(\frac{6}{n^3}\right)\right)^{n^3}\overset{1^\infty}{=}\lim_{x\to\infty}\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{x^3}=\exp\left(\lim_{x\to\infty}\ln\left[\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{x^3}\right]\right)$$

$$=\exp\left(\lim_{x\to\infty}x^3\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)\right)\overset{\infty\cdot 0}{=}\exp\left(\lim_{x\to\infty}\frac{\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{\frac{0}{0}}}{x^{-3}}\right)$$

$$\overset{\text{L'H}}{=}\exp\left(\lim_{x\to\infty}\frac{\frac{1}{1-\sin\left(\frac{6}{x^3}\right)}\cdot\left(-\cos\left(\frac{6}{x^3}\right)\right)^{1}\cdot\left(-18x^{-4}\right)^{-1}\cdot(-18x^{-4})^{6}}{-3x^{-4}}\right)=e^{-6}=\boxed{\frac{1}{e^6}}$$ Converges

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Examples Continue on Next Page

Example 5: $\left\{\dfrac{(3n-1)!}{(3n+1)!}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\frac{(3n-1)!}{(3n+1)!}=\lim_{n\to\infty}\frac{\cancel{(3n-1)!}}{(3n+1)(3n)\cancel{(3n-1)!}}=\lim_{n\to\infty}\frac{1}{(3n+1)(3n)}=\boxed{0}$$ (the denominator $(3n+1)(3n)\to\infty$, so the limit is 0) Converges

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Next, complete the following HW problems

Assigned Problems for HW 10

List the first five terms of the Sequence. (Start with $n=1$)

1. $a_n=\dfrac{(-1)^{n-1}}{5^n}$

2. $a_n=\dfrac{1}{(n+1)!}$

3. $a_n=\dfrac{(-1)^n\ n^2}{n+1}$

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit. Justify as always! No guessing.

4. $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$

5. $\left\{\dfrac{5n^2+3}{2n^2-7n}\right\}_{n=1}^{\infty}$

6. $\left\{\dfrac{3n^4-n-5}{7n^4+n^2-9}\right\}_{n=1}^{\infty}$

7. $\left\{\dfrac{\tan^{-1}n}{n}\right\}$

8. $\left\{\dfrac{n^2}{e^n}\right\}$

9. $\left\{n\sin\left(\dfrac{1}{n}\right)\right\}$

10. $\left\{\dfrac{(\ln n)^2}{n}\right\}_{n=1}^{\infty}$

11. $\left\{\dfrac{n^{99}}{\ln n}\right\}_{n=2}^{\infty}$

12. $\left\{\dfrac{\ln(99)}{n^{99}}\right\}$

13. $\left\{\left(1+\dfrac{1}{n}\right)^n\right\}_{n=1}^{\infty}$

14. $\left\{\left(1-\dfrac{5}{n^6}\right)^{n^6}\right\}_{n=1}^{\infty}$

15. $\left\{\left(1-\arcsin\left(\dfrac{3}{n^2}\right)\right)^{n^2}\right\}$

16. $\{\ln(2n^2+1)-\ln(n^2+1)\}$

17. $\left\{\dfrac{(n+3)!}{(n+1)!}\right\}_{n=1}^{\infty}$

18. $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}$

19. $\left\{\cos^2\left(\dfrac{\pi n^6+6}{6n^6+1}\right)\right\}_{n=1}^{\infty}$

20. $\left\{\arctan\left(\dfrac{5n^7+1}{5n^7+7}\right)\right\}_{n=1}^{\infty}$

My (Drop-In) Office Hours: SMUD 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30–3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow Evening Drop-in Hours: SMUD 207

Sunday	6:00–7:30pm:	**Natalie** Stott
Sunday	7:30–9:00pm:	**Oscar** Hernandez
Monday	6:00-7:30pm:	**Aaron** Cordoba
Monday	7:30–9:00pm:	**Oscar** Hernandez
Tuesday	6:00-7:30pm:	**Gretta** Ineza
Wednesday	7:30–9:00pm:	**Natalie** Stott
Thursday	6:00-7:30pm:	**Gretta** Ineza
Thursday	7:30–9:00pm:	**DJ** Beason
Friday	6:00-7:30pm:	**Aaron** Cordoba
Friday	7:30–9:00pm:	**DJ** Beason

• My Office Hours are times to drop in to my office, unannounced. Math Fellow hours are also for unannounced drop-ins, in SMUD 207, at the hours above.
All are welcome! Just stop by. Working on your calculus assignment can be fun! I encourage you to come hang out at many of these help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.