Homework #1

Due **Friday, September 6** in Gradescope by **11:59 pm ET**

- **WATCH** Video 2: *Integration Review*
- **READ** the eight worked-out examples in this handout
- **WRITE AND SUBMIT** solutions to the 20 assigned problems in this handout

NOTE: Show your work! Use the solutions to the eight worked examples here as guides for how much work to show on these sorts of problems.

INDEFINITE Integral Examples: Always remember to add $+C$ right away, as soon as you compute the *Most General Antiderivative*.

1. $\displaystyle\int e^x \cos\left(e^x\right)\, dx$ $\quad \boxed{\begin{array}{rl} u &= e^x \\ du &= e^x dx \end{array}} \quad = \displaystyle\int \cos u\, du = \sin u + C = \boxed{\sin\left(e^x\right) + C}$

---

2. $\displaystyle\int \frac{\cos x}{e^{\sin x}}\, dx$ $\quad \boxed{\begin{array}{rl} u &= \sin x \\ du &= \cos x dx \end{array}} \quad = \displaystyle\int \frac{1}{e^u}\, du = \int e^{-u}\, du = -e^{-u} + C$

$= \boxed{-e^{-\sin x} + C}$ or $= \boxed{-\dfrac{1}{e^{\sin x}} + C}$

---

3. $\displaystyle\int e^{2x}\left(4 - e^{2x}\right)^6\, dx$ $\quad \boxed{\begin{array}{rl} u &= 4 - e^{2x} \\ du &= -2e^{2x} dx \\ -\frac{1}{2}du &= e^{2x} dx \end{array}} \quad = \dfrac{1}{2}\displaystyle\int u^6\, du = -\frac{1}{2} \cdot \frac{u^7}{7} + C$

$= \boxed{-\dfrac{1}{14}(4 - e^{2x})^7 + C}$

---

4. $\displaystyle\int \frac{7}{e^{4x}\left(6 + e^{-4x}\right)}\, dx$ $\quad \boxed{\begin{array}{rl} u &= 6 + e^{-4x} \\ du &= -4e^{-4x} dx \\ -\frac{1}{4}du &= e^{-4x} dx \\ -\frac{1}{4}du &= \dfrac{1}{e^{4x}} dx \end{array}} \quad = -\dfrac{7}{4}\displaystyle\int \frac{1}{u}\, du$

$= -\dfrac{7}{4}\ln|u| + C = \boxed{-\dfrac{7}{4}\ln\left|6 + e^{-4x}\right| + C}$

Note: You can drop the absolute values, i.e. $\boxed{-\dfrac{7}{4}\ln\left(6 + e^{-4x}\right) + C}$ because $6 + e^{-4x} > 0$.

(continued next page)

DEFINITE Integral Examples: Remember, you must *change* or *mark* your limits of integration when you do substitution with a definite integral.

5. $\displaystyle\int_{\ln 2}^{\ln 6} \frac{e^x}{2+e^x}\,dx$ $\quad\boxed{\begin{array}{rl} u &= 2+e^x \\ du &= e^x\,dx \end{array}}$ and $\boxed{\begin{array}{ll} x=\ln 2 & \Longrightarrow u = 2+e^{\ln 2} = 2+2 = 4 \\ x=\ln 6 & \Longrightarrow u = 2+e^{\ln 6} = 2+6 = 8 \end{array}}$

$$= \int_4^8 \frac{1}{u}\,du = \ln|u|\Big|_4^8 = \ln 8 - \ln 4 = \ln\left(\frac{8}{4}\right) = \boxed{\ln 2}$$

---

6. $\displaystyle\int_e^{e^3} \frac{4}{x(\ln x)^2}\,dx$ $\quad\boxed{\begin{array}{rl} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{array}}$ and $\boxed{\begin{array}{ll} x=e & \Longrightarrow u = \ln e = 1 \\ x=e^3 & \Longrightarrow u = \ln(e^3) = 3 \end{array}}$

$$= \int_1^3 \frac{1}{u^2}\,du = 4\int_1^3 u^{-2}\,du = -4u^{-1}\Big|_1^3 = -\frac{4}{u}\Big|_1^3 = -\frac{4}{3} - (-4) = -\frac{4}{3} + \frac{12}{3} = \boxed{\frac{8}{3}}$$

---

7. Here is an example that might feel a bit confusing. Also, it's indefinite, so there's a $+C$!

$\displaystyle\int x\sqrt{x+1}\,dx$ $\quad\boxed{\begin{array}{rl} u &= x+1 \Rightarrow x = u-1 \\ du &= dx \end{array}}$ $\quad = \displaystyle\int (u-1)\sqrt{u}\,du$

$$= \int u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$$

---

8. Here is an example if you prefer to *mark* your limits of integration for substitution, instead of changing them. Generally, we will *change* the limits in class. But no matter what, it is *incorrect* to leave the limits unchanged when doing a substitution on a definite integral

$\displaystyle\int_{\pi/6}^{\pi/3} \tan x\,dx$ $\quad\boxed{\begin{array}{rl} u &= \cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{array}}$ $\quad = \displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin x}{\cos x}\,dx = -\int_{x=\pi/6}^{x=\pi/3} \frac{1}{u}\,du$

$$= -\ln|u|\Big|_{x=\pi/6}^{x=\pi/3} = -\ln|\cos x|\Big|_{\pi/6}^{\pi/3} = -\left(\ln\left|\cos\left(\frac{\pi}{3}\right)\right| - \ln\left|\cos\left(\frac{\pi}{6}\right)\right|\right)$$

$$= -\left(\ln\left(\frac{1}{2}\right) - \ln\left(\frac{\sqrt{3}}{2}\right)\right) = -\left((\ln 1 \overset{0}{-} \cancel{\ln 2}) - (\ln\sqrt{3} - \cancel{\ln 2})\right)$$

$$= -\left(-\ln\sqrt{3}\right) = \boxed{\ln\sqrt{3}} \quad \text{or} \quad = \boxed{\frac{1}{2}\ln 3}$$

Next, complete the following HW problems
found on the next page

Assigned Problems for HW 1

Please try to work these integrals quickly and efficiently, yet still fully justifying your solutions. (Soon, for simple substitutions like $u = 2x - 3$ we might start just seeing it in our heads; but **not yet**.) Besides using the examples in this homework packet as models, you can also see Section 4.5 (of Stewart's Calculus textbook) or the Integration Review video if needed. And of course, help is also available in Office Hours!

1. $\displaystyle\int \frac{1}{e^{7x}}\,dx$

2. $\displaystyle\int e^{14x}\,dx$

3. $\displaystyle\int e^{1-2x}\,dx$

4. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

5. $\displaystyle\int e^x \sin\left(e^x\right)\,dx$

6. $\displaystyle\int \cos\left(\frac{x}{5}\right)\,dx$

7. $\displaystyle\int \sec^2\theta \tan^3\theta\,d\theta$

8. $\displaystyle\int (2-3x)^5\,dx$

9. $\displaystyle\int \frac{1}{7-x}\,dx$

10. $\displaystyle\int \frac{1}{\sqrt{7x+5}}\,dx$

11. $\displaystyle\int \frac{1}{(3-5x)^2}\,dx$

12. $\displaystyle\int \frac{1}{2x-1}\,dx$

13. $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

14. $\displaystyle\int x\sqrt{7-3x^2}\,dx$

15. $\displaystyle\int \frac{1}{x\ln x}\,dx$

16. $\displaystyle\int \sin(\pi x+1)\,dx$

17. $\displaystyle\int x(1-x)^{79}\,dx$

18. $\displaystyle\int x^3(x+1)^{79}\,dx$

19. $\displaystyle\int \frac{x^2}{\sqrt{3-x}}dx$

20. $\displaystyle\int_0^{\ln 2} \frac{1}{e^{3x}\left(2-e^{-3x}\right)^2}dx$

- Don't forget to mark or change limits of integration on problem 20.
- You may need help on 17–20, so jump into Office Hours!
- Reminders about office hours, gradescope, and academic honesty on the next page...

MY (DROP-IN) OFFICE HOURS

Seeley Mudd 406

Tuesday: 1:30–3:00 pm

Thursday: 1:30-3:00 pm

Friday: 2:00–3:00 pm

(or by appointment)

Math Fellow evening TA Help Hours start soon; details TBA

• Please turn in your homework by the deadline. Gradescope will LOCK YOU OUT at 11:59 pm.

• Please **TAG** your HW solution numbers in Gradescope. Ask for help if needed.

• Write a **final** draft neatly in either pen or pencil or on a tablet. Arrange your work clearly on the page, and avoid messes and cramped writing. Don't write right up to any of the four edges of your paper; that is, **leave some margins at the edges**. Make sure that your scan is **clear and legible** for all of your work on all problems.

• You are responsible for writing up your own solutions, in your own words. Please read the discussion about the Statement of Intellectual Responsibility from our class syllabus. **No** use of online solutions, AI, other textbooks, or any other outside resources. I will give zero credit for all work copied from any other source. I will also report you to the Dean of Conduct/Community Standards. You will also risk an F in the course.

• **NO LATE HOMEWORK!** except in cases of incapacitating illness, family emergencies, or the like.