Math 121-03, Fall 2024 Professor Rob Benedetto

What you need to know for Exam 2

You should know Sections 7.4, 7.5, 7.8, and 11.1–11.7. The exam will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with exponentials, logarithms, inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

Remember, no calculators in any exams.

You may bring one standard size (8.5x11") "cheat sheet" of notes to the exam

No worked-out problems are allowed on your cheat sheet.

You must hand-write your own cheat sheet directly on the paper. No printouts.

At the end of the exam, you must hand in your cheat-sheet with your exam paper.

NOTE: With your exam, I will provide you with a fresh copy of the two-page "Summary of Convergence Tests" handout (from both class and on the website). So you may reserve your cheat sheet for other things, like tips and notes for how best to use those tests.

- 7.4: Partial Fractions, for Proper Cases only; that is, integrating rational functions with deg(numer) < deg(denom). (But also including Competing the Square, which technically is from Section 7.3.) That is, factor the denominator, solve for $A, B, \ldots$, etc. Then integrate the resulting pieces (usually, but not always, ln's and arctan's).
- 7.5: Integration Strategy. This section reminds you how to approach integrating most integrals we've seen, which of course could come up in any improper integral.
- 7.8: Improper Integrals. Be able to recognize improper integrals of either type I or II, and know how to compute them by turning them into limits of integrals. (Note: We did not cover the more complex *split* cases.) You may need L'H Rule to finish the Improper Limit.
- 11.1: Sequences. Know what sequences are, and be able to compute their limits (or determine that they diverge). Remind yourself how to use L'H Rule, but remember to switch from n to x before applying L'H to the limit of a sequence.
- 11.2: Series. Know what series are, and don't confuse them with sequences. Mainly learn Geometric series (including GST) and the n^{th} Term Divergence Test (nTDT).
- 11.3: Integral Test. Know the Integral Test (IT) and be able to use it. If asked, make sure you explicitly check all three of the requirements before applying it. Also know the p-Test.
- 11.4: Comparison Tests. Know the Comparison Test (CT) and the Limit Comparison Test (LCT). Choose the comparison (testing) series wisely, and make sure to check the requirements before invoking either test. Analyze the comparison series with fine detail. Be clear on your conclusion about the original series.
- 11.5: Alternating Series Test (AST). Know the definition of an alternating series, know how to recognize one, and know the AST. Know that if the terms do **not** go to 0, then you cannot use AST; but you *can* use the n^{th} Term Divergence Test (nTDT) in that case.
- 11.6: Absolute Convergence Test (ACT), Ratio Test (RT). Also know the definitions of Absolute Convergence (AC), Conditional Convergence (CC), and Divergence. If I ask whether a series converges absolutely, it does *not necessarily* mean that you use the Ratio Test. Instead, the Ratio Test works great for series involving exponentials (like e^n or 2^n), factorials, and/or superexponentials (n^n), even if there are smaller factors like polynomials multiplied in.

- 11.7: Strategy for Testing Series. The main theme: get used to using the Tests to decide convergence/divergence of a series even if the problem doesn't tell you which test to use.

Some things you don't need to know

- Sections 7.4/7.5: Long division of polynomials. Inverse hyperbolic functions.
- Section 7.8: Comparison Test for integrals (which is near the end of the section).
- Section 11.3–11.6: Proofs of the various convergence tests.
- Section 11.3–11.5: The stuff about estimating sums or remainders.
- Section 11.6: Root test, and Rearrangements of series.

Tips

- Practice, practice, practice; sometimes with classmates, sometimes at office hours or Fellow hours, and sometimes on your own. Do the review packet and check the answer keys (posted online). Take the practice tests, at least one or two under test conditions (timed, closed book). Spend time in advance preparing your cheat sheet; the process of writing things down is a good study method itself! Get started studying and practicing early, and then ask me and/or the Math Fellows about problems you were unable to do or unsure of.
- During the exam itself, be prepared to try several different things on any given integral or series. If you can't figure out a problem, skip it and come back to it later. But try not to just stare; keep trying to come up with different strategies, and keep writing stuff down.
- Improper Integrals: **Recognizing them**: a ∞ or $-\infty$ in the limits of integration is a dead giveaway, but also look to see whether the integrand has a vertical asymptote at either endpoint. **Computing them:** Compute the integral on each piece by taking a limit as t approaches the bad endpoint. The limit sign is important; **don't** leave it off.
- For sequences: know all the methods; but remember, if you want to use L'Hôpital, you must convert from the integer-only variable n to the real variable x.
- Make sure not to confuse a sequence with a series.
- Don't make up an "n^{th} term *convergence* test" or any other nonexistent tests.
- For series: try not to misuse a convergence test. The different tests have similar ingredients, and similar things to check, but they are *not* quite the same. Even though I will provide you with the (two-page) sheet of convergence tests during the exam, you should be doing enough practice and studying that you essentially know all the tests from memory, and are only using the sheet as a security blanket and a double-check.
- If you're asked for the **sum** of a series (as opposed to its convergence or divergence), that pretty much means that you must be dealing with a geometric series, or maybe the sum of two such things.
- Train being efficient in writing your convergence of series answers. Be explicit about:
  1. your convergence conclusion or declaration,
  2. your test(s) used (or type of series), **and**
  3. write the reasons or show all of the conditions checked.

  Use words, and don't omit any details or steps!