Math 121 Midterm Exam #2 April 11, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [36 Points] Compute the following **Improper** Integrals. Justify your work.

(a) $\displaystyle\int_{-4}^{-3} \frac{6}{x^2+2x-8}\, dx$ You can use this **free** Partial Fractions fact:

$$\frac{6}{(x-2)(x+4)} = \frac{1}{x-2} - \frac{1}{x+4}$$

(b) $\displaystyle\int_{-\infty}^{0} \frac{6}{x^2+2x+4}\, dx$

(c) $\displaystyle\int_{0}^{1} \ln x\, dx$ You can use this **free** fact $\displaystyle\int \ln x\, dx = x\ln x - x + C$

(d) $\displaystyle\int_{0}^{e^5} \frac{1}{x\ [25+(\ln x)^2]}\, dx$

2. [20 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6^{2n}} + \frac{\ln(2022)}{n^6}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{2022}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{n^6}{\ln(n+2022)}$

3. [8 Points] Name any convergence test(s) you use, and justify all of your work.

Use the Absolute Convergence Test to prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6+2022}$ Converges.

4. [36 Points] In each case Determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+6}{n^6+2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 6^n \cdot n!}{n^6 \cdot n^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2022}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Prove that the sequence $\displaystyle\left\{ \frac{(\ln n) \cdot 2^n \cdot (n!)^2}{n^{2n} \cdot (3n)!} \right\}_{n=1}^{\infty}$ Converges.