Math 121 Midterm Exam #2 October 30, 2019

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute the following integrals. Justify your work.

(a) $\displaystyle\int_0^{e^3} \frac{1}{x\ [9+(\ln x)^2]}\, dx$

(b) $\displaystyle\int_0^1 x\ln x\ dx$

(c) $\displaystyle\int_7^9 \frac{10}{x^2-8x-9}\, dx$

(d) $\displaystyle\int_7^\infty \frac{10}{x^2-8x+19}\, dx$

2. [8 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n}{n+1}\right)^n \right\}_{n=1}^{\infty}$$

3. [8 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n\, \frac{3^{2n-1}}{4^{2n+1}}$$

4. [20 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{\sin^2(n^3+1)}{n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e} + \frac{1}{e^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{e}{n^e} + \frac{1}{e^n}$

5. [24 Points] Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+9}{n^9+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (2n)! \, \ln n}{(n^n) \, n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{1}{n+6}$

****

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1 Compute the sum for this series $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n+2} - e^{2n}}{(e^{2n}+1)\,(e^{2n+2}+1)}$

OPTIONAL BONUS #2 It can be shown that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$

Compute the following sum. Justify. $\displaystyle 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \dots$