

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.
- Numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $e^{\ln 4}$ ,  $\ln(e^7)$ ,  $e^{3\ln 3}$ ,  $\sinh(\ln 3)$ , or  $\arctan(\sqrt{3})$  should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] For each of the following integrals, either compute it or show that it diverges.

(a)  $\int_5^{\infty} \frac{1}{x^2 - 6x + 13} dx$

(b)  $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$

(c)  $\int_1^2 \frac{1}{x^2 - x} dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left( 1 + \ln \left( 1 + \frac{5}{n} \right) \right)^n \right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n \frac{4^{2n+1}}{3^{3n-1}}$$

4. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)  $\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{\sin^2(5n)}{n^5 + 3}$

5. [25 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)  $\sum_{n=1}^{\infty} (-1)^n \frac{3n^5 + 6n^3}{n^9 + 4}$

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 2^{4n} n^n}{(3n)! \ln n}$

(c)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n} + 1}$

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## OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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**OPTIONAL BONUS #1** Compute the sum of the following series

$$\sum_{n=2}^{\infty} \frac{e^{2n+2} - e^{2n}}{(e^{2n} + 1)(e^{2n+2} + 1)}$$

**OPTIONAL BONUS #2** Compute the following integral  $\int \frac{x^5 + 7x^3 + x^2 + 13x + 2}{x^4 + 6x^2 + 9} dx$ .