Practice Problems for Midterm Exam 2

Instructions: The point of this set of practice problems is **NOT** that you should plan to do all of these problems; there are **WAY** too many. Instead, the point is that you should skip around and try various different types of problems. And if you find you could use more practice with a particular type of problem, you should be able to find several more like it here.

So don't try to do all of these problems. But try to do a lot of them — a broad variety of them, but also extras on any particular topics that you find you could use the most practice on.

Differentiation Rules Differentiate the following functions. Simplify your answers unless otherwise specified.

1. $y = \sin^3(x^3)$
2. $y = \cos^2(3x)$
3. $f(t) = t^2 \sin^5(2t)$
4. $H(x) = \left(1 - \dfrac{2}{x^2}\right)^5$
5. $f(x) = \sqrt[3]{x^3+8}$
6. $g(t) = \dfrac{t^3 + \tan\left(\frac{1}{t}\right)}{1+t^2}$
7. $p(x) = \dfrac{1}{(-2x+3)^5}$
8. $r(x) = \dfrac{(2x+1)^3}{(3x+1)^4}$
9. $S(x) = \left(\dfrac{1+2x}{1+3x}\right)^4$
10. $g(x) = \cos(3x)\sin(4x)$
11. $g(x) = (x + 7x^{-6})\sqrt{2x+1}\cos^2(6x)$ (Do not simplify much)
12. $g(x) = \dfrac{\cos(3x)}{\sin(4x)}$
13. $w = \dfrac{5(1+x^2)^3}{x\sqrt{2x+1}}$
14. $y = \left((x^2+3x)^4 + x\right)^{-5/7}$ (Do not simplify much)
15. $g(t) = \cos\left(\sin^3\left(\dfrac{t}{\sqrt{t+1}}\right)\right)$
16. $g(x) = \cos^2(6x)\left(\dfrac{\tan x}{\sqrt{2x+1}}\right)$

Differentiation Rules Compute the following derivatives, and simplify your answers. (In some cases you may want to simplify **before** differentiating, but I won't give you that hint on the exam.)

17. $k'(x)$, where $k(x) = \dfrac{1}{x} + 5x^3$.
18. $H'(x)$, where $H(x) = (x + \sqrt{x^4+1})^5$
19. y', where $\dfrac{x}{y+1} = x^2 - y^2$.
20. $\dfrac{dv}{dx}$, where $v(x) = \dfrac{x^2+2x-7}{\sqrt[3]{x}}$.
21. $f'(x)$, where $f(x) = \sqrt[3]{x^2 + \tan x}$
22. $r'(x)$, where $r(x) = x^3\sec^2(5x-3)$
23. $\dfrac{dh}{dt}$, where $h(t) = (3t-1)^7(4t+3)^9$
24. $\dfrac{dF}{dy}$, where $F(y) = y^2\cos\sqrt{y}$
25. $\dfrac{dG}{du}$, where $G(u) = \dfrac{3u^2}{u^3 - 5\sin u}$

More Derivatives:

26. Find the second derivative of each of the following functions. (Hint: you might save some work by simplifying. And I won't give you that hint on the test.)

$$f(x) = \frac{x^2 + 3x - 7}{\sqrt{x}} \qquad g(t) = \frac{t^5 - t^3}{t^2 - 1} \qquad h(x) = x\sqrt{x^2 - 4}$$

27. Let f and g be two differentiable functions, and suppose that their values and the values of their derivatives at $x = 1, 2, 3$ are given by the following table:

x	1	2	3
$f(x)$	3	2	5
$f'(x)$	-2	1	3
$g(x)$	3	1	4
$g'(x)$	-3	2	7

Let $h(x) = f(g(x))$ and $k(x) = f(x) \cdot g(f(x))$. Compute $h'(2)$ and $k'(1)$.

28. Let f and g be two differentiable functions, and suppose that their values and the values of their derivatives at $x = 2, 3$ are given by the following table:

x	2	3
$f(x)$	4	0
$f'(x)$	1	-7
$g(x)$	3	-1
$g'(x)$	-5	4

Let $h(x) = f(x)g(x), k(x) = \dfrac{f(x)}{g(x)}$, and $W(x) = f \circ g(x)$. Compute $h'(2)$ and $k'(2)$ and $W'(2)$.

29. Let $f(x) = \sqrt{x+1} \cdot g(x)$ where $g(0) = -7$ and $g'(0) = 4$. Compute $f'(0)$.

30. Let $f(x) = \dfrac{\sqrt{x^2+1}}{g(x)}$ where $g(0) = -7$ and $g'(0) = 4$. Compute $f'(0)$.

More Implicit Differentiation: For each of the equations described below, find an equation of the tangent line to the curve at the given point.

31. $x^3 + x^2y + 4y^2 = 6$ at $(1, 1)$.

32. $4(x + y)^2 = x^2y^2$ at $(-2, 1)$.

33. $\dfrac{x}{y+1} = x^2 - y^2$ at $(1, 0)$.

34. $4\cos x \sin y = 3$ at $(\pi/6, \pi/3)$.

35. $y^3 - xy^2 + \cos(xy) = 2$ at $(0, 1)$

Related Rates:

36. A rowboat is in the water near a dock, and a rope attached to the bow of the boat is connected to a winch on the dock. The winch is 6 ft above the water, and the bow of the boat is 1 ft above the water. The winch pulls the boat towards the dock by retracting 1 ft of rope per second, keeping the rope taut. When the boat is 12 feet from the dock, how fast is it moving towards the dock?

37. A hot circular plate of metal is cooling. As it cools its radius is decreasing at the rate of 0.01 cm/min. At what rate is the plate's area decreasing when the radius equals 50 cm?

38. A child is flying a kite at a height of 300 feet above the level of the child's hand. The kite is being blown by the wind so that it is moving horizontally at 25 ft/sec. Assuming that the string is always a straight line from the child's hand to the kite, how fast is the child letting out string at the moment when the kite is 500 ft from the child?

39. Suppose a snowball remains spherical while it melts, with the radius shrinking at one inch per hour. How fast is the volume of the snowball decreasing when the radius is 2 inches?

40. A kite 100 feet high is being blown horizontally at 8 feet per second. When there are 300 feet of string out:

(a) how fast is the string running out?

(b) how fast is the angle between the string and the horizontal changing?

41. A 6 foot tall man walks with a speed of 8 feet per second away from a street light that is atop an 18 foot pole. How fast is the top of his shadow moving along the ground when he is 100 feet from the light pole?

42. Two trucks leave a depot at the same time. Truck A travels east at 40 miles per hour, while Truck B travels north at 30 miles per hour. How fast is the distance between the trucks changing 60 minutes after leaving the depot?

43. Suppose a spherical balloon is inflated at the rate of 10 cubic inches per minute. How fast is the radius of the balloon increasing when the radius is 5 inches?

44. A child riding in a car driving along a straight road is looking through binoculars when she sees a water tower off to the side. The tower is located 1500 ft from the nearest point on the road. At a particular moment, the car is moving at 80 feet per second, and the car is 800 feet from that nearest point to the tower. How fast must the child be rotating the angle that the binoculars are pointing to keep the tower in view?

45. A photographer is televising a 100-meter dash from a position 5 meters from the track in line with the finish line. When the runners are 12 meters from the finish line, the camera is turning at the rate of $\frac{3}{5}$ radians per second. How fast are the runners moving then?

Absolute Extrema: Find the absolute maximum and absolute minimum values of the following functions on the given intervals.

46. $f(x) = 3x^{2/3} - \dfrac{x}{4}$ on $[-1, 1]$.

47. $h(x) = \dfrac{x^2 - 1}{x^2 + 1}$ on $[-1, 3]$.

48. $F(x) = -2x^3 + 3x^2$ on $[-\frac{1}{2}, 3]$.

49. $f(x) = x^{2/3}$ on $[-1, 8]$.