Practice Test A for Midterm Exam 3

Instructions: This optional exam is for practice, to give you an idea of what our in-class midterm exam will be like. I'd recommend that you try taking it in exam conditions: 50 minutes, closed-book.

---

1. **(20 points)** You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

2. **(25 points)** Compute $\displaystyle\int_1^3 x^2 - 3x \; dx$ using each of the following **two** different methods:

(a) The Fundamental Theorem of Calculus.

(b) Riemann Sums and the limit definition of the definite integral.

3. **(30 points)** Let $\quad f(x) = \dfrac{-x^2 + x + 2}{x^2 - 2x + 1} = \dfrac{-x^2 + x + 2}{(x-1)^2}.$

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve $y = f(x)$.

You may take my word for it that:

$$f'(x) = \frac{x-5}{(x-1)^3} \qquad \text{and} \qquad f''(x) = \frac{-2x+14}{(x-1)^4}$$

4. **(10 points)** Compute the following limits.

(a) $\displaystyle\lim_{x \to \infty} \frac{x^2 - x + 5}{3x^7 + x^6 - 2022}$

(b) $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{4x^2 + 5x}}{6x + 7}$

5. **(15 points)** Compute the following definite and indefinite integrals.

(a) $\displaystyle\int \frac{y^2 - 4y + 1}{\sqrt{y}} \, dy$

(b) $\displaystyle\int_{-\pi/4}^{5\pi/6} 3x + \cos x \, dx$