

Practice Test A for Midterm Exam 1

Instructions: This optional exam is for practice, to give you an idea of what our in-class midterm exam will be like. I'd recommend that you try taking it in exam conditions: 50 minutes, closed-book.

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow -3} \frac{x^2 - 2x - 15}{x^2 + x - 6} =$

(b) $\lim_{x \rightarrow 5} \frac{x^2 - 2x - 15}{|5 - x|} =$

(c) $\lim_{x \rightarrow 2} \frac{x^2 - 2x - 15}{x^2 + x - 6} =$

(d) $\lim_{x \rightarrow 5} \frac{x^2 - 2x - 15}{x^2 + x - 6} =$

(e) $\lim_{x \rightarrow 2} \frac{x + 7}{(x - 2)^2} =$

(f) $\lim_{x \rightarrow -1} \frac{H(x + 1) - H(-1 - x)}{x + 1} =$ where $H(x) = \sqrt{x + 2}$

2. [13 Points] Use translation, etc. to graph the following two functions:

$$f(x) = 5 + \sqrt{6 - x}$$

$$g(x) = \frac{1}{10}(x + 2)^2$$

3. [15 Points] Suppose that $f(x) = \frac{x + 7}{x - 3}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x) = x^2 - 7x - 12$. Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = -2$. ****Use the limit definition of the derivative when computing the derivative.****

5. [12 Points] Suppose that f and g are functions, **and**

• $\lim_{x \rightarrow 7} f(x) = 5$ • $\lim_{x \rightarrow 7} g(x) = -3$ • $f(5) = 7$ • $g(x)$ is continuous at $x = 7$.

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a) $\lim_{x \rightarrow 7} \sqrt{3f(x) - 7g(x)} =$

(b) $\lim_{x \rightarrow 7} \frac{f(x)}{1 - x} =$

(c) $g(7) =$

(d) $g \circ f(5) =$

(problems continue next page)

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\ 1 & \text{if } x = 3 \\ 6 - 2x & \text{if } 0 < x < 3 \\ 16 - x^2 & \text{if } -4 < x \leq 0 \\ \frac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

$$\text{(c) Compute } \begin{cases} \lim_{x \rightarrow 0^+} f(x) = \\ \lim_{x \rightarrow 0^-} f(x) = \\ \lim_{x \rightarrow 0} f(x) = \end{cases} \quad \text{(d) Compute } \begin{cases} \lim_{x \rightarrow 3^+} f(x) = \\ \lim_{x \rightarrow 3^-} f(x) = \\ \lim_{x \rightarrow 3} f(x) = \end{cases}$$

$$\text{(e) Compute } \begin{cases} \lim_{x \rightarrow -4^+} f(x) = \\ \lim_{x \rightarrow -4^-} f(x) = \\ \lim_{x \rightarrow -4} f(x) = \end{cases}$$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.