

### Midterm Exam 1

*Instructions:* Do all six numbered problems. If you wish, you may also attempt the optional bonus problems. You may use one 8.5x11" sheet of notes that you brought with you for reference during the exam. **You must justify your answers**, both on computational problems and on proof problems. Simplify answers whenever possible. Answers such as  $7 + 3\pi\sqrt{5}$  or  $55/15$  or  $\sin(\pi/7)$  are acceptable as final answers. If you are not sure whether or not some argument or statement requires further justification, please ask me about it. If you need more space, use the back of any page. If you have time, check your answers.

**SILENCE AND PUT AWAY ALL CELL PHONES AND DEVICES**  
**WRITE LEGIBLY. NO CALCULATORS.**

1. **(34 points)** Evaluate each of the following limits. Please **justify** your answers. Be clear about whether the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a)  $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{x^2 - 2x - 8}$

(b)  $\lim_{x \rightarrow -7} \frac{x^2 + 2x - 35}{|x + 7|}$

(c)  $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{(x - 3)^3}$

(d)  $\lim_{x \rightarrow 1} \frac{2 - \sqrt{x + 3}}{x^2 - 1}$

2. **(8 points)** Use translation, etc. to graph the function  $f(x) = 4 - \sqrt{x - 2}$

3. **(16 points)** Let  $f(x) = \frac{x + 7}{x - 3}$ . Compute  $f'(x)$  using the **limit definition of the derivative**.

4. **(14 points)** Let  $g(x) = x^2 - 7x - 4$ . Find an equation of the tangent line to the curve  $y = g(x)$  at the point where  $x = -1$ .

Use the **limit definition of the derivative** when differentiating.

5. **(10 points)** Let  $f$  and  $g$  be functions such that

$$\lim_{x \rightarrow 4} f(x) = -2 \quad \lim_{x \rightarrow 4} g(x) = 3 \quad g(-2) = 4 \quad f \text{ is continuous at } x = 4$$

Evaluate the following, and **justify your answers**. (Justifications can be brief, but they can't be completely absent.)

(a)  $\lim_{x \rightarrow 4} (3f(x) + (g(x))^2)$

(b)  $f(4)$

(c)  $f \circ g(-2)$

6. **(18 points)** Let  $f(x) = \begin{cases} \frac{1}{x-1} & \text{if } x < 2, \\ (x-3)^2 & \text{if } x \geq 2. \end{cases}$

(a) What is the domain of  $f$ ? (And briefly justify, of course.)

(b) Compute:  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$ , and  $\lim_{x \rightarrow 2} f(x)$

(c) Carefully sketch the graph  $y = f(x)$ .

(d) State the value(s) at which  $f$  is discontinuous. Justify your answer(s) briefly.

**OPTIONAL BONUS A. (2 points.)** Let  $g(x) = \sqrt{x^3 - 4x^2 - 7}$ . Compute  $g'(x)$  using the **limit definition of the derivative**.

**OPTIONAL BONUS B. (1 point.)** There are five countries that are permanent members of the United Nations Security Council. Name them.