Math 111-01, Fall 2023 Professor Rob Benedetto

**Some Basic Functions**

Real-life functions are complicated and impossible to get exact formulas for. Examples include:

- $f(t) =$ average global temperature at time $t$.
- $g(x) =$ number of live individual bacteria in the petri dish at time $x$.
- $a(p) =$ number of years person $p$ will live to be.
- $s(d) =$ closing price per share of stock in the company on day $d$.

So to understand these sorts of real-life functions, we usually try to come up with approximations that are not exactly correct but have the advantage of having **explicit formulas** that we can actually compute with. And to make those explicit formulas we need some ingredients, i.e., some basic functions that we can combine with one another to make more complicated explicit functions. This handout gives a short list of some of the basic functions we'll use in Math 111.

- **Linear functions**: These are functions of the form $f(x) = mx + b$, where $m$ and $b$ are constants.

  Specific examples include $f(x) = 5x - 2$, or $g(x) = -\frac{\sqrt{2}}{\pi} \cdot x$, or $h(x) = 3$.

  The graph of $f(x) = mx + b$ is the straight line $y = mx + b$ of slope $m$ and $y$-intercept $b$.

- **Polynomials**: These are functions of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $n \geq 0$ is an integer (called the *degree* of $f$, provided $a_n \neq 0$), and each coefficient $a_i$ is a constant.

  For example, $g(x) = 3x^4 - \pi x^2 + x - \frac{7}{2}$ is a polynomial of degree 4.

  All linear functions $f(x) = mx + b$ are polynomials. You should also have some idea of how to graph degree 2, or *quadratic* polynomials $h(x) = ax^2 + bx + c$, whose graphs $y = ax^2 + bx + c$ are parabolas (provided $a \neq 0$, of course).

  You don't (yet) need to know how to graph polynomials of degree 3 or more.

- **Power functions**: These are functions of the form $f(x) = x^a$, where $a$ is a constant.

  When $a$ is a nonnegative integer, like $a = 3$, then $f(x) = x^3$ is also a polynomial.

  But when $a$ is a fraction, it means to take roots. For example, $x^{1/2}$ means $\sqrt{x}$, and $x^{1/3}$ means $\sqrt[3]{x}$, and in general, $x^{1/n}$ means $\sqrt[n]{x}$, the (positive) $n$-th root of $x$.

  More generally, $x^{m/n}$ means $\sqrt[n]{x^m}$, which is the same as $(\sqrt[n]{x})^m$.

  And a negative exponent means take the reciprocal. So $x^{-2} = \dfrac{1}{x^2}$, and $x^{-2/3} = \dfrac{1}{\sqrt[3]{x^2}}$.

  You should familiarize yourself with the graphs of various power functions. We'll review them as they arise in various examples in class.

- **Rational functions**: These are functions of the form $f(x) = \dfrac{g(x)}{h(x)}$ where $g$ and $h$ are both polynomials, like $f(x) = \dfrac{5x^2 - 7x + 2}{3 - x}$. Don't worry about trying to graph them (yet).

- **Trigonometric functions**: These are the six specific functions

  $$\sin x, \quad \cos x, \quad \tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}, \quad \sec x = \frac{1}{\cos x}, \quad \text{and } \csc x = \frac{1}{\sin x}.$$

  You should have seen some trig before, but don't worry, we'll review along the way.